Chaos, Solitons & Fractals 59 (2014) 129–134

Contents lists available at ScienceDirect

Chaos, Solitons & Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos

Cyclic organization of stable periodic and chaotic pulsations in Hartley's oscillator

Joana G. Freire [a,b], Jason A.C. Gallas [a,b,c,d,*]

[a] Departamento de Física, Universidade Federal da Paraíba, 58051-970 João Pessoa, Brazil
[b] Centro de Estruturas Lineares e Combinatórias, Faculdade de Ciências, Universidade de Lisboa, 1749-016 Lisboa, Portugal
[c] Institute for Multiscale Simulations, Friedrich-Alexander-Universität Erlangen-Nürnberg, 91052 Erlangen, Germany
[d] Instituto de Altos Estudos da Paraíba, Rua Infante Dom Henrique 100-1801, 58039-150 João Pessoa, Brazil

ARTICLE INFO

Article history:
Received 25 November 2013
Accepted 20 December 2013

ABSTRACT

A recent conjecture in this Journal, concerning the existence of spiral stability phases in Hartley's oscillator, is corroborated amply. We report numerically computed stability phase diagrams indicating precisely where spirals of periodicity and chaos may be found in several control planes of the system. In addition, we describe some remarkable parameter loops in control space which allow one to trace identical dynamical behaviors by tuning totally independent parameters.

1. Introduction

Electronic circuits containing nonlinear elements have a long tradition in exhibiting rich dynamical behaviors as already described in several books [1–4]. However, despite all knowledge accumulated so far, researchers have not been idle and nonlinear circuits remain subjects of much activity and sources of surprising and unanticipated results. The present work is motivated by interesting and intriguing findings reported recently in this Journal by Tchitnga et al. [5], who were able to find chaos experimentally in Hartley's oscillator (Fig. 1).

As it is known, Hartley's oscillator is a classical device introduced in 1915, based on two coils and one capacitor. Later, in 1918, Colpitts introduced his celebrated circuit based on two capacitors and one coil which, therefore, is the electrical dual of Hartley's oscillator. Both oscillators were key components in early radio telephony and still remain standard devices that can be used to produce a wide range of frequencies [6]. Since such oscillators are governed by three first-order ordinary differential equations, one can expect them to generate chaotic oscillations.

Remarkably, while Colpitts' oscillator has been extensively investigated for its chaotic [7] as well as its hyperchaotic [8] dynamics, similar investigations for Hartley's oscillator remain essentially nonexistent to this date. Notable exceptions are the timely work of Tchitnga et al. [5], and an earlier work by Kvarda [9]. Since the oscillators of Hartley and Colpitts are duals, it is theoretically interesting to investigate if such duality is in some way reflected also in their stability diagrams.

In parallel developments, the presence of certain remarkable points, called periodicity hubs [10,11] have been recently reported in simulations and experiments [12,13] of the behavior of an autonomous circuit discussed in this Journal by Kyprianidis et al. [14], Koliopanos et al. [15], and Stoupoulos et al. [16]. Such hubs are attracting attention nowadays because they act as remarkable organizing centers from which an infinite number of spirals of stable oscillatory phases emanate. Based on certain similarities between the equations of motion governing the two-component circuit of Kyprianidis et al. and those underlying Hartley's oscillator, it was conjectured [12] that Hartley's oscillator could also display hubs and spirals of the type found in the two-component circuit.

The aim of this paper is to report a numerical investigation that amply corroborates the presence of infinite families of spiral phases of stability in phase diagrams for

* Corresponding author at: Departamento de Física, Universidade Federal da Paraíba, 58051-970 João Pessoa, Brazil.
E-mail address: jason.gallas@cbi.uni-erlangen.de (J.A.C. Gallas).

0960-0779/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.chaos.2013.12.007

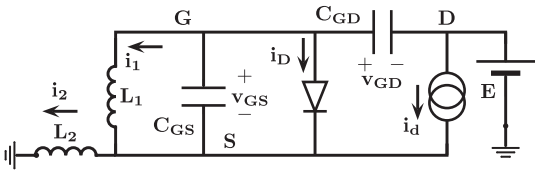

Fig. 1. Schematic representation of Hartley's oscillator, governed by the four-dimensional flow in Eqs. (1)–(4).

Hartley's oscillator. In addition, we also report other amazing circular structures which allow one to follow identical dynamical behaviors by tuning totally independent parameters. Such circular structures were found for a multitude of sections of the multi-dimensional control parameter space. Since Hartley's oscillator involves a relatively high number of parameters, the experimental search of its spirals and other structures can be prohibitively time-consuming. Here, the parameter plane characterization was done thanks to the decisive help of 1536 high-performance processors of a SGI Altix cluster with a theoretical peak performance of 16 Tflops.

2. Hartley's circuit and its autonomous flow

As mentioned, the circuit studied here (Fig. 1) contains a JFET and a tapped coil and was investigated experimentally by Tchitnga et al. [5]. To model the circuit, these authors considered some simplifications, namely, they neglected the internal resistance of the tapped coil and used a high-frequency small-signal equivalent circuit model of a JFET, as described in their paper. In this case, the equations governing the circuit are the following [5]:

$$C_{GS}\frac{dv_{GS}}{dt} = -i_1 + i_2 - i_D - i_d, \tag{1}$$

$$C_{GD}\frac{dv_{GD}}{dt} = -i_2 + i_d, \tag{2}$$

$$L_1\frac{di_1}{dt} = v_{GS}, \tag{3}$$

$$L_2\frac{di_2}{dt} = -v_{GS} + v_{GD} + E, \tag{4}$$

where the currents are

$$i_d = \begin{cases} 0, & \text{if } v_{GS} \leqslant V_c, \\ g(v_{GS} - V_c)^2, & \text{if } v_{GD} \leqslant V_c, \\ g(v_{GS} - v_{GD})(v_{GS} + v_{GD} - 2V_c), & \text{if } v_{GD} \geqslant V_c, \end{cases}$$

$$i_D = I_S[exp(v_{GS}/V_T) - 1].$$

All variables and parameters are defined in Fig. 1. Unless otherwise stated, we follow the experiments and fix $C_{GS} = 3.736$ pF, $C_{GD} = 3.35$ pF, $I_S = 33.57$ fA, $V_c = -1.409$ V, $V_T = 25$ mV, $E = 2.8$ V, $g = 1.754$ mAV^{-2}, $L_1 = 24.5$ μH and $L_2 = 4$ μH.

As a preliminary result and check of the model equations, Fig. 2 presents bifurcation diagrams which corroborate the chaotic region discovered by Tchitnga et al. [5]. In their Fig. 3, it seems that x_1 should read x_2. Here and below, computations were started at the minimum value of the parameter from the arbitrarily chosen initial condition $(v_{GS}, v_{GD}, i_1, i_2) = (-1.25, -2.5, 10^{-6}, 10^{-6})$ and continued by "following the attractor" [17]. The next Section reports our main findings, namely numerically obtained phase diagrams displaying stability regions of the self-pulsations generated by Hartley's oscillator or, equivalently, two-parameter bifurcation diagrams for the circuit.

3. Stability diagrams for Hartley's oscillator

Fig. 3 presents in two complementary ways (described below) phase diagrams characterizing the far-reaching regular organization induced by the set of stable oscillations of the circuit. Although obtained using two very distinct algorithms, the boundaries between chaotic and periodic regions match perfectly. What is more important, both phase diagrams reveal unambiguously the presence of periodicity hubs (focal points [10]) with a wide-ranging clockwise spiral organization around them.

Fig. 3(a) shows a Lyapunov stability diagram, obtained by plotting on a fine parameter grid the largest non-zero Lyapunov exponent. Such exponents are familiar indicators that allow one to discriminate chaos (positive exponents) from periodic oscillations (negative exponents) [18,19]. Fig. 3(b) presents an "isospike diagram" [20] namely, a diagram obtained by counting the number of peaks (local maxima) contained in one period of the periodic oscillations of a variable of interest, v_{GD} in all our diagrams. These latter diagrams are particularly helpful because, in addition to discriminating periodicity from chaos, they simultaneously display how the waveform of every periodic oscillation evolves as parameters are changed.

Each individual panel in Figs. 3–6 displays the analysis of self-pulsations recorded for a mesh of $1200 \times 1200 = 1.44 \times 10^6$ parameter points. They were obtained by integrating numerically Eqs. (1)–(4) using the standard fourth-order Runge–Kutta algorithm with fixed time-step $h = 5 \times 10^{-11}$. In all diagrams, integrations were always started from the same initial condition: $(v_{GS}, v_{GD}, i_1, i_2) = (-1.25, -2.5, 10^{-6}, 10^{-6})$. As usual, the first 2×10^5 integration steps were disregarded as a transient time needed to approach the attractor, with the subsequent 4×10^6 steps used to compute the Lyapunov spectrum. To discriminate solutions and to count the number of peaks within a period of v_{GD}, after computing the exponents we continued integrations for another 4×10^6 time-steps recording up to 800 extrema (maxima and minima) of v_{GD} and checking whether pulses repeated or not. The computation of stability diagrams is a standard calculation that we performed as described in detail, e.g., in Ref. [21].

Fig. 4(a)–(d) presents isospike diagrams as recorded for a distinct section of the control parameter space. This figure illustrates with increasing resolution the presence of periodicity hubs (focal points) with their characteristic infinite set of spirals. In contrast to what happens in Fig. 3, here the winding occurs anticlockwise around the main focal point [10]. The regular behavior revealed by the isospike diagrams in Fig. 4(a)–(d) was corroborated independently by simultaneously computing Lyapunov stability diagrams, one of them being shown in Fig. 4(e). The agreement is excellent.

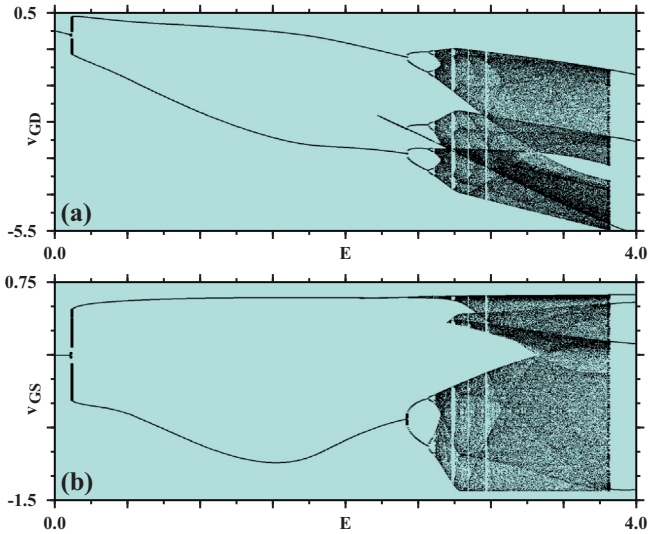

Fig. 2. Bifurcation diagrams of the local extrema for the variables (a) v_{GD} and (b) v_{GS}, showing the presence of chaotic solutions in the interval 2.6 V $< E <$ 3.7 V. All voltages are measured in Volts. Both axis are divided in 600 equally spaced points.

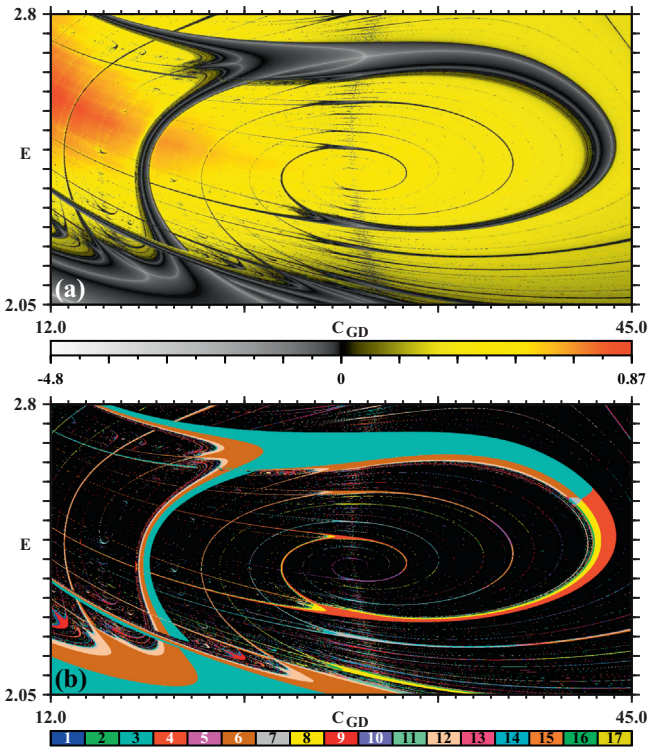

Fig. 3. Phases diagrams illustrating in two complementary ways infinite sequences of wide ranging clockwise spiraling phases of periodic oscillations. (a) Lyapunov stability diagram; (b) isospike diagrams displaying the number of peaks in one period of v_{GD}. Absence of periodicity, "chaos", is represented in black. See text. Capacitances are measured in pF, E in Volts.

In all isospike diagrams, we used a palette of 17 colors. Solutions having more than 17 peaks were plotted by recycling the 17 basic colors "modulo 17", namely by assigning them a color-index given by the remainder of the integer division of the number of peaks by 17. Multiples of 17 are given the index 17. In Fig. 4(a)–(c), black is used

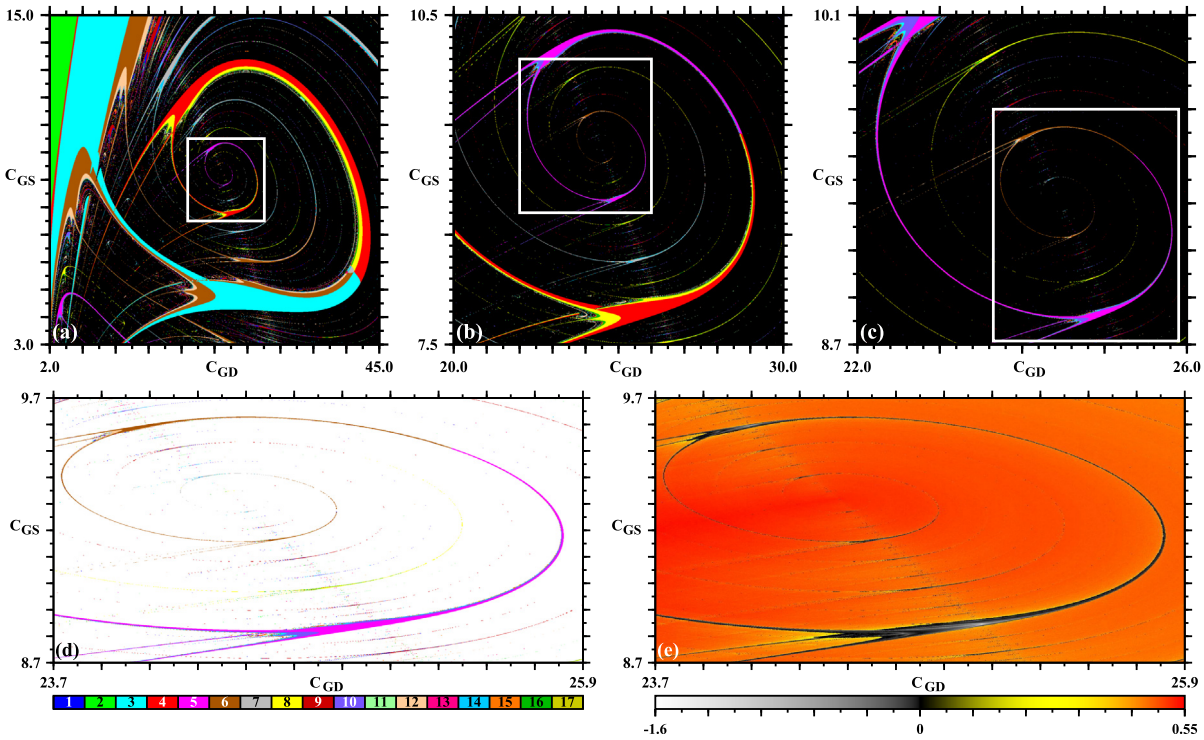

Fig. 4. Phases diagrams illustrating sequences of wide anticlockwise spiraling phases due to periodic oscillations. (a–d) Isospike diagrams displaying successive magnifications of the distribution of the number of peaks in one period of v_{GD}. Absence of periodicity, "chaos", is represented in black. (e) Lyapunov stability diagram corroborating the structure seen in panel (d). Here, absence of periodicity is shown in white. See text. Capacitances are in pF.

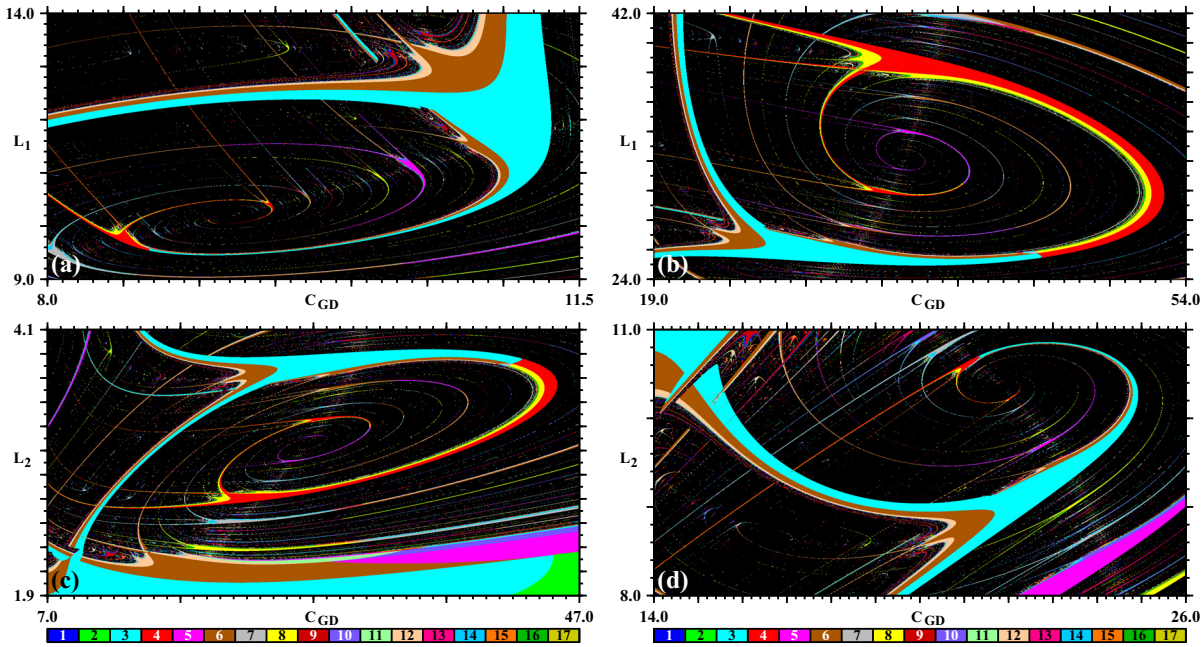

Fig. 5. Distinct parameter planes illustrating the two possible spiraling modes. Left column: clockwise spirals; Right column: anticlockwise spirals. Capacitors are measured in pF and inductances in μH.

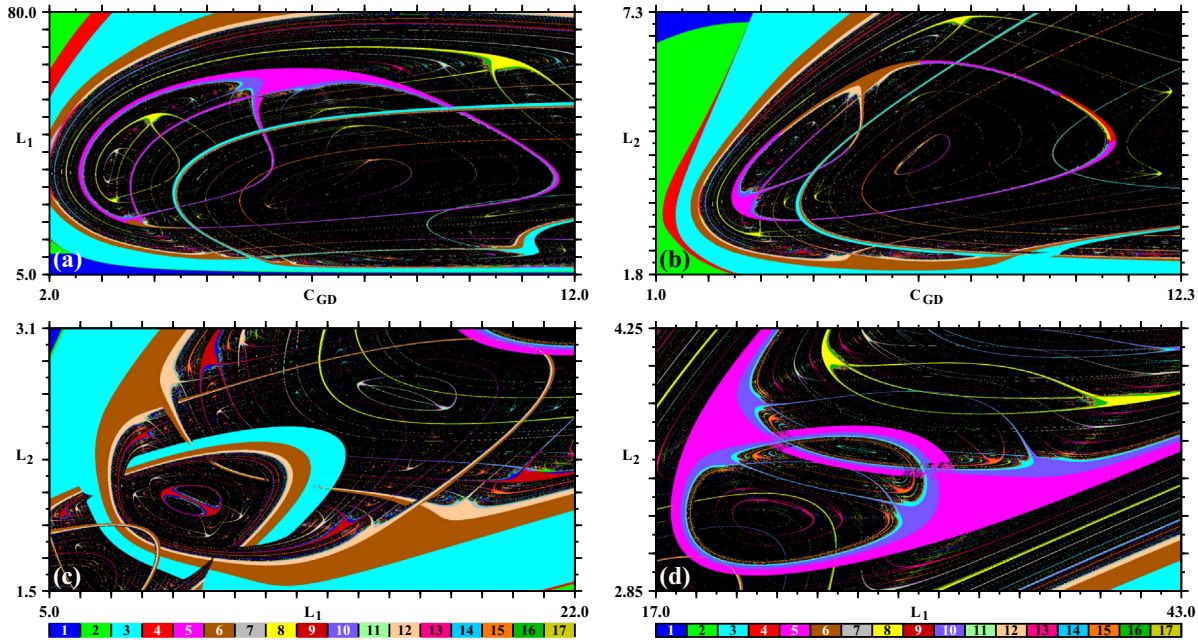

Fig. 6. Two distinct sections of the control parameter space illustrating characteristic circular structures formed by the interconnection of a pair of shrimps [23–25]. Such complex circular stability regions exist abundantly in all parameter sections investigated. Closed interconnections involving more than two shrimps can be also identified. Inductances are measured in μH and capacitors in pF.

to represent "chaos" (i.e., lack of numerically detectable periodicity). Under a different light, in Fig. 4(d) white is used to represent chaotic pulsations. From these figures one sees that the number of peaks contained in one period of the periodic oscillations of $v_{GD}(t)$ increases steadily by 1 after every turn towards the focal hub. It is perhaps of interest to mention that there is no special reason for taking peaks mod 17. This specific number is simply a good compromise for contrasting many oscillations with distinct number of peaks. Any other value (not too small) could also be used. The point of the palette is to try to maximize contrast between the profusion of small adjacent regions.

Fig. 5 displays a surprising behavior seen for spirals found for yet another set of control parameter planes. In these planes we found parameter windows capable of displaying simultaneously both clockwise and anticlockwise spirals. Such striking abundance of spirals was recently observed in a rather distinct system namely, in a circuit containing a tunnel diode [22]. It is important to recall that so far there is no theoretical framework to explain why such spiraling reversals can occur. Thus, Hartley's oscillator corroborates the behavior found for tunnel diodes and is a second example of an apparently complex effect waiting for a theoretical explanation. In all spirals, the number of peaks increases by one unit after a full turn, roughly, towards the focal hub. Similar results are obtained when using any of the other three variables to count peaks. But the key point here is simpler: the immense abundance of wide-ranging spirals in Hartley's oscillator. That the dual Colpitts oscillator also contains hubs and spirals profusely can be inferred from figures in the beautiful work by de Feo and Maggio [7].

As a final result, Fig. 6 presents for two different parameter planes another unexpected feature discovered in Hartley's oscillator. This figure shows several examples of pairs of "mating" shrimps, namely of shrimps [23–25] that are united perfectly by their four sets of "legs". In Fig. 6(b) there is a closed loop formed by four shrimps. Such circular shrimp arrangements exist in two distinct flavors: displaying either just a single color or being multicolored. This latter feature shows that the number of peaks within a period of v_{GD} changes when circling around the shrimp legs. Similar results emerge when counting peaks for any of the other variables of the problem. Again, as above, there is no theoretical underpinning to explain/predict such remarkable behaviors in control parameter plane. We remark that such circular arrangements are potentially interesting for experimental applications because they allow one to experience identical dynamical changes when tuning two rather distinct parameters.

4. Conclusions

In summary, this work augmented significantly the knowledge about chaotic pulsations in Hartley's oscillator by identifying a plethora of chaotic phases in several two-parameter stability diagrams. The conjecture concerning the existence of stability spiral phases in Hartley's oscillator was found to be true, with large spirals existing abundantly. The oscillator supports both clockwise and anticlockwise spirals, making them particularly attractive for further investigations. Hartley's circuit also shows certain circular arrangements of pairs of shrimps and many other complex structures that are not easy to classify

systematically. Cyclic arrangements involving more than two shrimps were also found abundantly. We believe the high-resolution phase diagrams presented here to be an useful asset for experimental work because they indicate parameter windows where to look for rich dynamics. Regrettably, so far there is no theoretical prescription to locate spirals or any other of the stability phases discussed here. Thus, the only way to find them is either by direct numerical prospection or through experimental work. We hope that this work may stimulate the experimental verification of the self-generated pulsation phases for Hartley's oscillator. This system has an exceptionally rich dynamics that certainly harbors many useful surprises.

Acknowledgements

JGF was supported by FCT, Portugal through the Post-Doctoral grant SFRH/BPD/43608/2008. JACG was supported by CNPq, Brazil, and by the Deutsche Forschungsgemeinschaft through the Cluster of Excellence *Engineering of Advanced Materials*. All computations were done in the CESUP-UFRGS clusters in Porto Alegre, Brazil.

References

[1] Carroll TL, Pecora LM. Nonlinear dynamics in circuits. Singapore: World Scientific; 1996.
[2] Ogorzalek MJ. Chaos and complexity in nonlinear electronic circuits. Singapore: World Scientific; 1997.
[3] Chen G, Ueta T, editors. Chaos in circuits and systems. Singapore: World Scientific; 2002.
[4] Ling BW, Iu HH, Lam H. Control of chaos in nonlinear circuits and systems. Singapore: World Scientific; 2008.
[5] Tchitnga R, Fotsin HB, Nana B, Fotso PHL, Woafo P. Chaos Solitons Fractals 2012;45:306.
[6] Millman J, Halkias C, Parikh CD. Millman's integrated electronics. New York: McGraw Hill; 2009.
[7] de Feo O, Maggio GM. Int J Bifurcation Chaos 2003;13:2917 [and several references therein].
[8] Cenys A, Tamasevicius A, Baziliauskas A, Krivickas R, Lindberg E. Chaos Solitions Fractals 2003;17:349.
[9] Kvarda P. Int J Bifurcation Chaos 2002;12:2229.
[10] Vitolo R, Glendinning P, Gallas JAC. Phys Rev E 2011;84:016216.
[11] Barrio R, Shilnikov A, Shilnikov LP. Int J Bifurcation Chaos 2011;22:1230016.
[12] Cabeza C, Briozzo CA, Garcia R, Freire JG, Marti A, Gallas JAC. Chaos Solitons Fractals 2013;52:59.
[13] Freire JG, Cabeza C, Marti A, Pöschel T, Gallas JAC. Sci Rep 2013;3:1958.
[14] Kyprianidis IM, Stoupoulos IN, Haralabidis P, Bountis T. Int J Bifurcation Chaos 2000;8:1903.
[15] Koliopanos CL, Kyprianidis IM, Stoupoulos IN, Anagnostopoulos AN, Magafas L. Chaos Solitons Fractals 2003;16:173.
[16] Stoupoulos IN, Miliou AN, Valaristos AP, Kyprianidis IM, Anagnostopoulos AN. Chaos Solitons Fractals 2007;33:1256.
[17] Freire JG, Field RJ, Gallas JAC. J Chem Phys 2009;131:044105.
[18] Kautz R. Chaos: the science of predictable random motion. Oxford: Oxford University Press; 2011.
[19] Tél T, Gruiz M. Chaotic dynamics: an introduction based on classical mechanics. Cambridge: Cambridge University Press; 2006.
[20] Freire JG, Gallas JAC. Phys Chem Chem Phys 2011;13:12191 [Phys Lett A 2011; 375:1097].
[21] Sack A, Freire JG, Lindberg E, Pöschel T, Gallas JAC. Sci Rep 2013;3:3350.
[22] Francke RE, Pöschel T, Gallas JAC. Phys Rev E 2013;87:042907.
[23] Gallas JAC. Physica A 1994;202:196.
[24] Bonatto C, Gallas JAC, Ueda Y. Phys Rev E 2008;77:026217.
[25] Lorenz EN. Physica D 2008;237:1689.