ABSORPTIVE OPTICAL BISTABILITY WITH

LASER AMPLITUDE FLUCTUATIONS

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It is generally believed that in optical bistability realistic fluctuations of the injected laser signal can influence the bistable behavior of the system in an important way. As an illustration of this general statement, we present in this paper the case of absorptive optical bistability (AOB) with random Gaussian fluctuations of the driving electric field amplitude.

For such external fluctuations, the microscopic part of the dynamical system can be described by a set of simple macroscopic state equations which, after a reduction of some of the degrees of freedom, can be converted into a single non-linear state equation which shows explicitly the bistable behavior of the system.

The theory of AOB in the limit of low-transmission mirrors and weak enough absorption gives the following well known relation between the dimensionless transmitted x and the incident electric field y:

$$\frac{dx}{dt} = -(x + \frac{2Cx}{1+x^2}) + y \qquad (1)$$

where the time is measured in units of the cavity bandwidth $\kappa$ and C is the order parameter of the system.

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C. M. Bowden et al. (eds.), *Optical Bistability 2*

A partially coherent laser with a finite bandwidth results in a non-white additive noise in this non-linear macroscopic state equation.

For a laser operating far above threshold the total electric field amplitude y(t) consists of two components: a coherent part $A_o$ and a small fluctuation $\delta y(t)$ which is Gaussian with the following mean value and correlation function:

$$\langle \delta y(t) \rangle = 0 \qquad \langle \delta y(t)\,\delta y(t') \rangle = a \exp\left(-\frac{|t-t'|}{b}\right) \tag{2}$$

where the dimensionless parameters a and b describe the characteristic properties of the fluctuating laser amplitude. The parameter $b = \kappa \cdot \tau_c$ is the coherence time $\tau_c$ of the laser amplitude fluctuations in units of the cavity linewidth and the coefficient a measures the intensity of the noise which in most realistic experiments is $a \leq 0.1$, i.e. a better than 10% stabilization of the amplitude can be obtained.

With such amplitude fluctuations the AOB equation (1) takes the form of the following nonlinear Langevin equation with an additive Orsnstein-Uhlenbeck stochastic process:

$$\frac{dx}{dt} = F(x) + \delta y(t) \tag{3}$$

where F(x) is the deterministic part of the dynamical evolution 1 given by:

$$F(x) = -\left(x + \frac{2Cx}{1+x^2}\right) + A_o \ . \tag{4}$$

In Ref. 1 we have established for the stochastic equation (4) a proper Fokker-Planck equation in the limit of large laser linewidth $(b < 1)$ and good stabilization of the amplitude fluctuations $(a < 1)$. For times larger than the transients $t \sim b$, this Fokker-Planck equation for the probability distribution of the transmitted field takes the following form:[1]

$$\frac{\partial}{\partial t} P = - \frac{\partial}{\partial x} F \cdot P + D \frac{\partial^2}{\partial x^2} K \cdot P \tag{5}$$

where the nonconstant diffusion function K(x) has the following form:

$$K(x) = (1 + b\,F'(x)) = 1 - b - 2bC \frac{1-x^2}{(1+x^2)^2} \tag{5}$$

and where $D = a \cdot b$.

In the white-noise limit, i.e. if $b \to 0$ with $D = ab$ = constant, we have $K \to 1$ and the diffusion term takes the well-known constant form. In this case D plays the role of the diffusion constant. In general, i.e. for $b \neq 0$, the diffusion function 6 depends on the laser linewidth b. This dependence is shown explicitly in Figure 1.

In order to describe a proper Fokker-Planck equation, the diffusion function given by Eq. (6) must be positive. This condition is fulfilled for all values of x only if $b < 1$, i.e. for values of b for which Eq. (5) should hold.

From the Fokker-Planck equation (5) we derive the following form of the stationary solution ($\frac{\partial}{\partial t} P_{st} = 0$), assuming natural boundary conditions:

$$P_{st} = N \exp\left(-\frac{U(x)}{D}\right) \tag{7}$$

where

$$U(x) = -\int dx \frac{F(x)}{1+bF'(x)} + ab \ \ell n\left|1+bF'(x)\right| \tag{8}$$

and N is a normalization constant.

States of maximal probability are characterized by the absolute minima of the thermodynamical potential U(x). In Figure 7 we have shown the form of this thermodynamical potential for various values of the incident field $A_o$ and for two values of b. It is clear from these curves that the depth and the width of the bistable minima depend on the laser parameters a and b.

The most probably values of $P_{st}$ given by Eq. (7) lead to the following steady-state relation ($U'(x) = 0$):

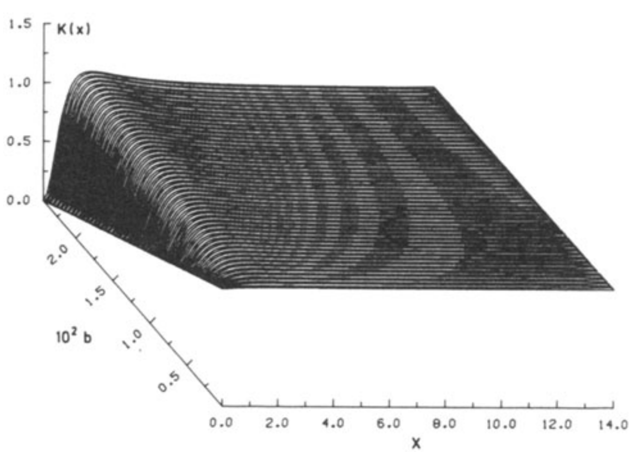


Fig. 1. Curves of the diffusion function K(x) for various values of x and laser bandwidth b.

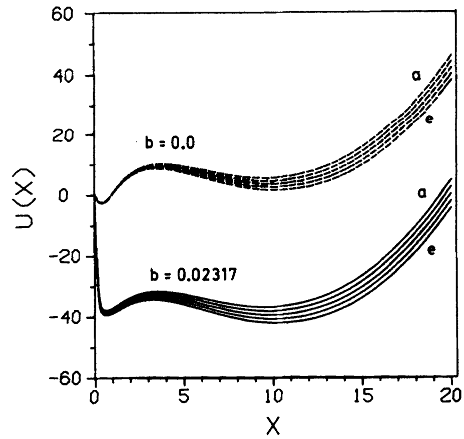


Fig. 2. These two sets of curves show the shape of the thermodynamical potential U(x) for two values of b and for the input field $A_o$ equal to : 13.7, 13.8, 13,9, 14.0, 14.1 (a-e). The dotted lines correspond to the white-noise case given by b=0. All these curves were calculated for C=20 and a=0.4.

$$A_o = x + \frac{2Cx}{1+x^2} + 4ab^2 C \frac{3-x^2}{(1+x^2)^3} \tag{9}$$

Eq. (9) can be regarded as a generalization of the deterministic bistability condition for the case of laser amplitude fluctuations.

The bistable behavior of the system with laser amplitude fluctuations can be illustrated in Fig. 3 where we have plotted the stationary probability given by Eq. (7) for different values of the coherent laser field $A_o$.

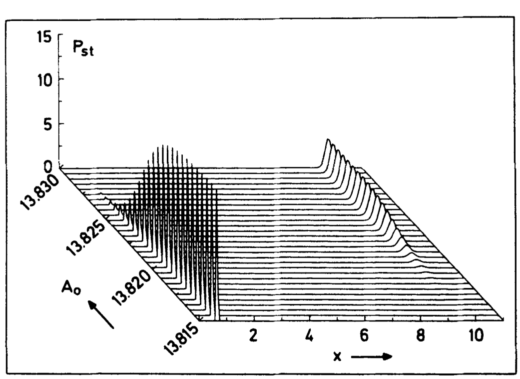


Fig. 3. Development of $P_{st}(x)$ as a function of the injected laser amplitude $A_o$ for C=20, a=0.4, and b= 0.02317.

As in the case of quantum fluctuations[2] the random amplitude of the laser field leads to a small range of values of $A_o$ in which the two peaks have a comparable area. Clearly the mean value of the transmitted field will coincide with one of the two deterministic branches except in this narrow transition region where we have large fluctuations.[1]

## ACKNOWLEDGMENTS

One of the authors (K.W.) would like to thank Professor H. Walther for his invitation to the Max-Planck Institute of Quantum Optics where a large part of this work has been done. Many discussions with Dr. P. Meystre are also acknowledged.

## REFERENCES

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2. See P. D. Drummond, this volume and references therein.