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MINIMIZING STOCHASTICITY IN THE NAO INDEX

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We show that the monthly North Atlantic Oscillation index routinely used in climatology as an indicator for global climate variability is not an optimal choice. A critical Markov analysis of the two pressure time-series for both, monthly and daily NAO index, indicates that the monthly index, due to its low sampling rate, contains higher stochastic terms than the daily index. Applying a recently developed variationally optimized Markov analysis leads to a new NAO index with minimal stochasticity.

Keywords: North Atlantic Oscillation; coupled Langevin equations; variational Markov analysis.

1. Introduction

The North Atlantic Oscillation (NAO) is a source of variability in the global atmosphere, describing a large-scale vacillation in atmospheric mass between the anticyclone near the Azores and the cyclone near Iceland [Wanner *et al.*, 2001; Hurrel, 1995]. It is receiving much attention in climate research because of its recently known importance in global climate variability. See [Wanner *et al.*, 2001] for a review on NAO. The spatial pattern of the NAO is a pronounced dipole-like pressure anomaly over the North Atlantic, with one pole at the Azores High and another over the Iceland Low. This dipole has two phases: a positive NAO phase, when there is a strong pressure gradient between both systems, and a negative phase, when the pressure gradient gets weaker. Lately, the question whether the NAO is a chaotic or a stochastic process was studied [Stephenson *et al.*, 2000] and it has been claimed that the NAO index is close to a Gaussian distribution [Collette & Ausloos, 2004]. The state of the NAO is usually measured by an index $\mathcal{N}$, defined as the normalized pressure difference between the high and the low poles, where the pressures are averaged over each month or year [Hurrel, 1995], yielding a time series assumed to be stationary [Stephenson *et al.*, 2000].

In this paper, we focus on the question whether a new NAO index with minimal stochasticity can be defined from the available pressure time series.

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Our analysis is based on a variationally optimized Markov analysis which has been recently introduced [Lind *et al.*, 2005] which provides a novel possibility to minimize stochasticity in coupled systems of Langevin equations. Since the daily index has a larger correlation length than the monthly index, this optimization procedure, applied to the daily data, yields an index with higher predictive power.

Using the Markov analysis described in [Friedrich *et al.*, 1997a; Renner *et al.*, 2001] we will extract a Langevin equation from the NAO indices time-series. The numerical procedure is as follows. For a general time-series $\{X(n)\}$ with $n = 1, \ldots, N$, one considers the auxiliary time series $\{X_t(n)\} \equiv \{X(n+t)\}$ with $n = 1, \ldots, N-t$ and t an integer, and their probability density functions (PDF) $p(X, t)$. Assuming that the process is Markovian, the two first Kramers–Moyal coefficients (KMC) are computed, namely the drift coefficient $D^{(1)}$ and the diffusion $D^{(2)}$

$$
\begin{aligned}
D^{(k)}(X_\tau, \tau) &= \frac{1}{k!} \lim_{\Delta\tau \to 0} M^{(k)}(X_\tau, \tau, \Delta\tau) \\
&\simeq \frac{1}{k!} \lim_{\Delta\tau \to 0} \frac{1}{\Delta\tau} \sum_{X_{\tau+\Delta\tau}} (X_{\tau+\Delta\tau}(n) \\
&\quad - X_\tau(n))^k p
\end{aligned} \tag{1}
$$

with $k = 1, 2$. Here, $M^{(k)}(X_\tau, \tau, \Delta\tau)$ are the conditional moments, where the sum is taken over the bin discretization of the PDF of $X_{\tau+\Delta\tau}$ and $p \equiv p(X_{\tau+\Delta\tau}, \tau + \Delta\tau | X_\tau, \tau)$ is the conditional probability, extracted directly from the auxiliary time-series. As done in other contexts [Friedrich *et al.*, 1997b] with the aim to focus on the intermittent deviations of the turbulent regimes associated with the NAO, we consider a rescaling of time t labeling the values of the time-series into $\tau = \log_2(\ell_M / t)$ with ℓ_M being the Markov length [Friedrich *et al.*, 2000; Renner *et al.*, 2001; Risken, 1984]. Under certain conditions [Risken, 1984], $D^{(1)}$ and $D^{(2)}$ describe the deterministic and stochastic dynamics, respectively, yielding a Langevin equation

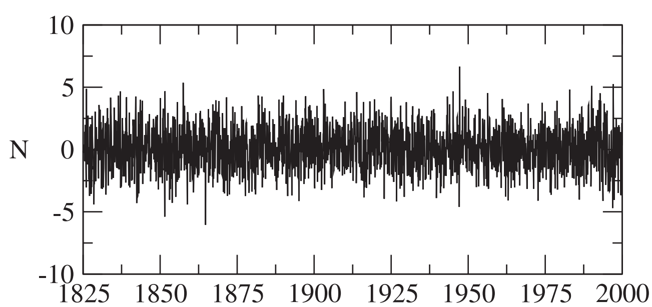
$$
\frac{d}{d\tau} X_\tau = D^{(1)}(X_\tau, \tau) + \eta(\tau)\sqrt{D^{(2)}(X_\tau, \tau)}, \tag{2}
$$

where $\eta(\tau)$ is a δ-correlated Gaussian noise.

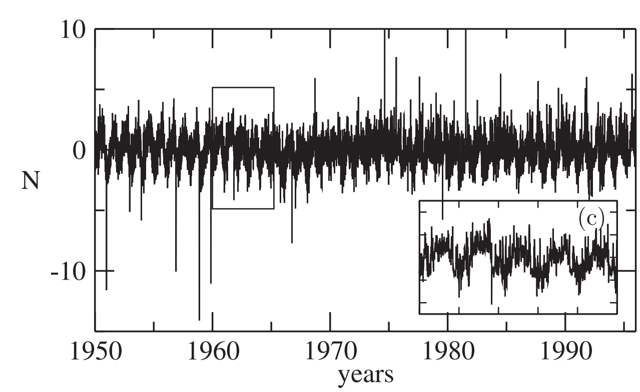
We start by determining the Langevin equation in Sec. 2 for both the monthly and the daily indices. In Sec. 3, we discuss the existence and amplitude of the dynamical and measurement noise present in such time-series and describe an analytical procedure to obtain optimal indices, i.e. with smaller stochastic terms. Conclusions are given in Sec. 4.

2. The Monthly and Daily NAO Indices as Stochastic Processes

In Fig. 1 the time series of the monthly [Fig. 1(a)] and daily [Figs. 1(b) and 1(c)] NAO indices are shown. As one sees, while the monthly index fluctuates in an apparently random way, the daily index shows a yearly modulation [see Fig. 1(c)]. In fact, as shown in Fig. 2, the correlation length for the daily index is about 100 days, while for the monthly index is approximately 1 month, i.e. close to one sampling step. Accordingly, the power spectrum of the monthly index shows a typical white noise spectrum, which is not the case of the daily index. These are first indications that the monthly index is hardly suitable as a time-series from which one can extract



(a)



(b)

Fig. 1. Time-series of two NAO indices: (a) monthly (January 1825 till November 2002) and (b) daily (1st Jan. 1950 till 30th Dec. 1996). The data was extracted from `http://www.cru.uea.ac.uk/cru/data/nao.htm` and `http://www.cgd.ucar.edu/cas/jhurrell/indices.html` respectively. In (c) a smaller section of the daily time-series is shown displaying the yearly modulation.

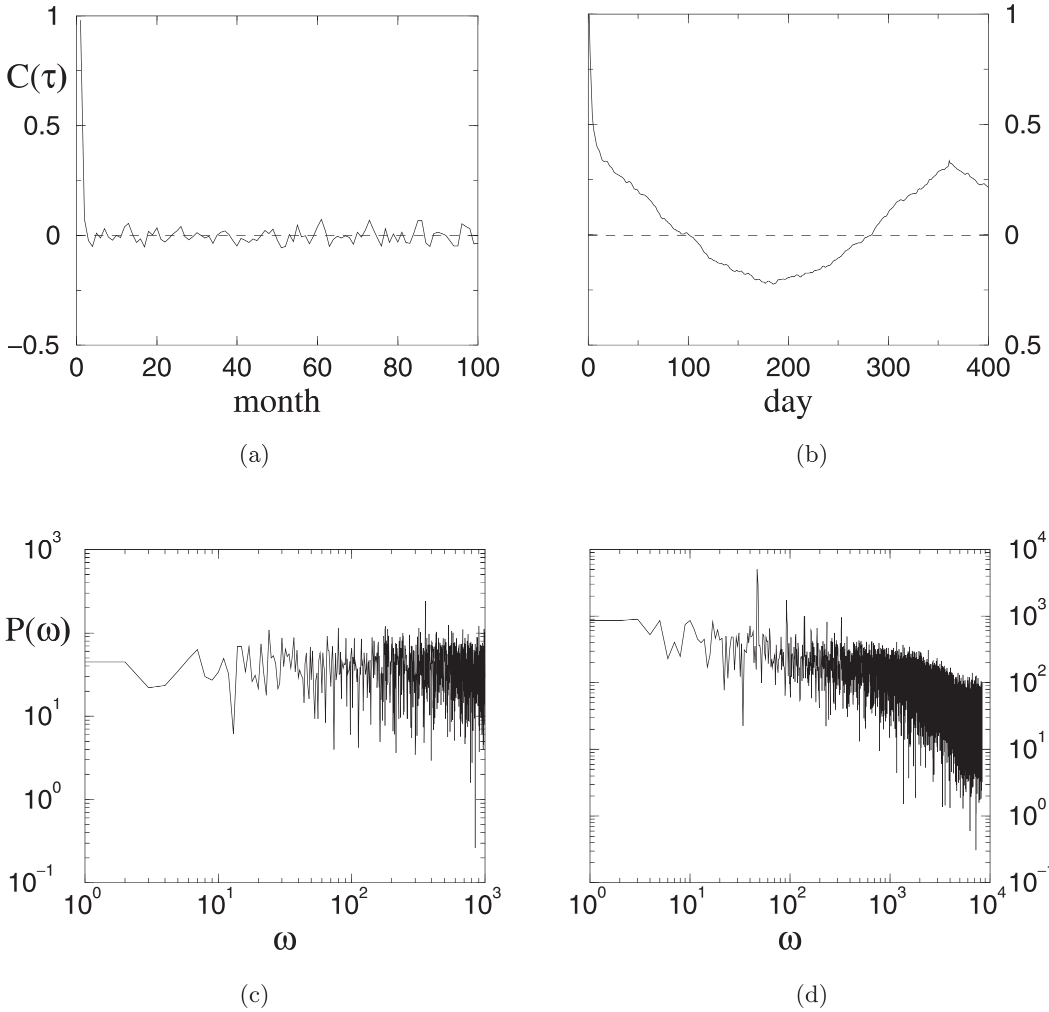


Fig. 2. The correlation function for (a) the monthly and (b) the daily NAO indices. The Fourier spectra for the monthly and the daily data are shown in the plots (c) and (d), respectively. Clearly, the daily data has a much larger correlation length than the monthly data which is close to white noise.

a significantly large deterministic component of the NAO system.

Computing the PDFs as described above, the conditional moments $M^{(1)}$ and $M^{(2)}$ in Eq. (1) can be studied as functions of the time difference $\Delta t = t_{\rm ref} - t$, where $t_{\rm ref}$ is the maximum value of t considered. Figure 3 shows the two conditional moments of the daily index for the reference $t_{\rm ref} = 90$ at three different arguments of its PDF. For small Δt the conditional moments behave in an irregular fashion, while for $\Delta t \gtrsim 20$ they vary approximately linearly with the time-lag. Therefore, $\ell_M = 20$ is assumed as the Markov length for the time-series, as indicated by the vertical dotted line in Fig. 3. Afterward we will confirm that indeed this was a proper choice.

Using the rescaled time-lag $\Delta\tau = \tau - \tau_{\rm ref} = \log_2(t_{\rm ref}/t)$ and making a linear fit of the conditional moments, $M^{(1)}$ and $M^{(2)}$, beyond the Markov length, intersecting it with the vertical axis $\Delta t = 0$, yields the limit in Eq. (1), leading to approximate values of the corresponding drift $D^{(1)}$ and diffusion $D^{(2)}$ coefficients respectively, as shown in Fig. 4.

Taking the same units as the daily NAO index, one clearly sees that, within the range $\mathcal{N}_\tau \in [-1, 1]$, $D^{(1)}$ varies linearly with $\mathcal{N}_\tau$, while $D^{(2)}$ varies quadratically. Fitting such curves yields

$$D^{(1)}_{\rm day}(\mathcal{N}_\tau) = 0.0629 - 0.8219\mathcal{N}_\tau, \tag{3}$$

$$D^{(2)}_{\rm day}(\mathcal{N}_\tau) = 1.2927 - 0.0364\mathcal{N}_\tau + 0.3300\mathcal{N}_\tau^2. \tag{4}$$

Both KMCs depend only weakly on the reference time $t_{\rm ref}$ as can be seen from Fig. 5. For both coefficients, the fitted surfaces yield general quadratic forms where the terms depending on $t_{\rm ref}$ are two orders of magnitude smaller than the terms depending on $\mathcal{N}_\tau$ alone.

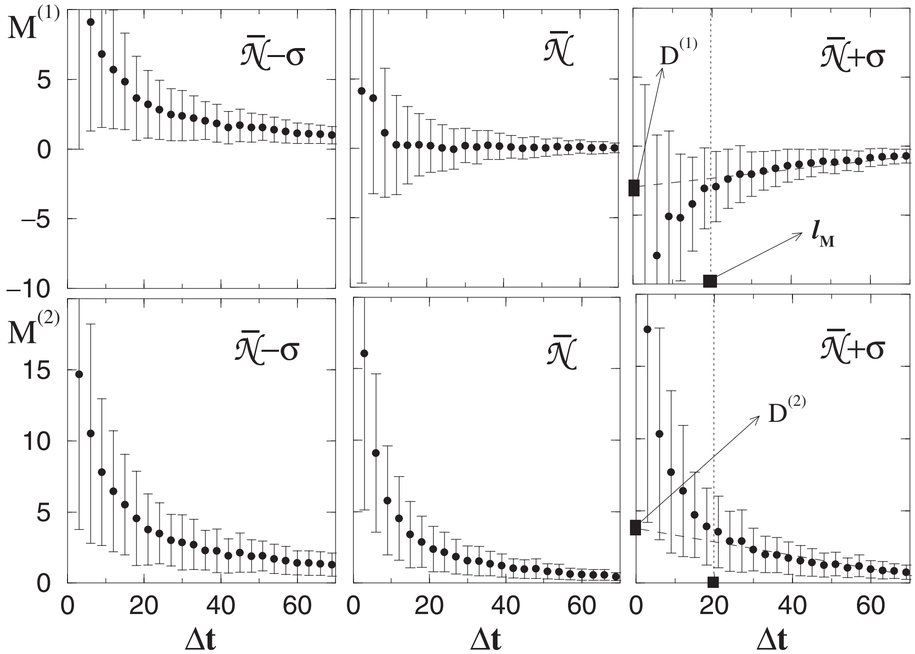


Fig. 3. The first and second conditional moments, M_1 and M_2 in Eq. (1) of the daily time-series shown in Fig. 1(b). Similar plots are obtained for the monthly data. Here, $t_{\text{ref}} = 90$. Fitting the curves beyond the Markov length $\ell_M = 20$ and intersecting the fits with the axis $\Delta t = 0$ yields the corresponding Kramers–Moyal coefficient.

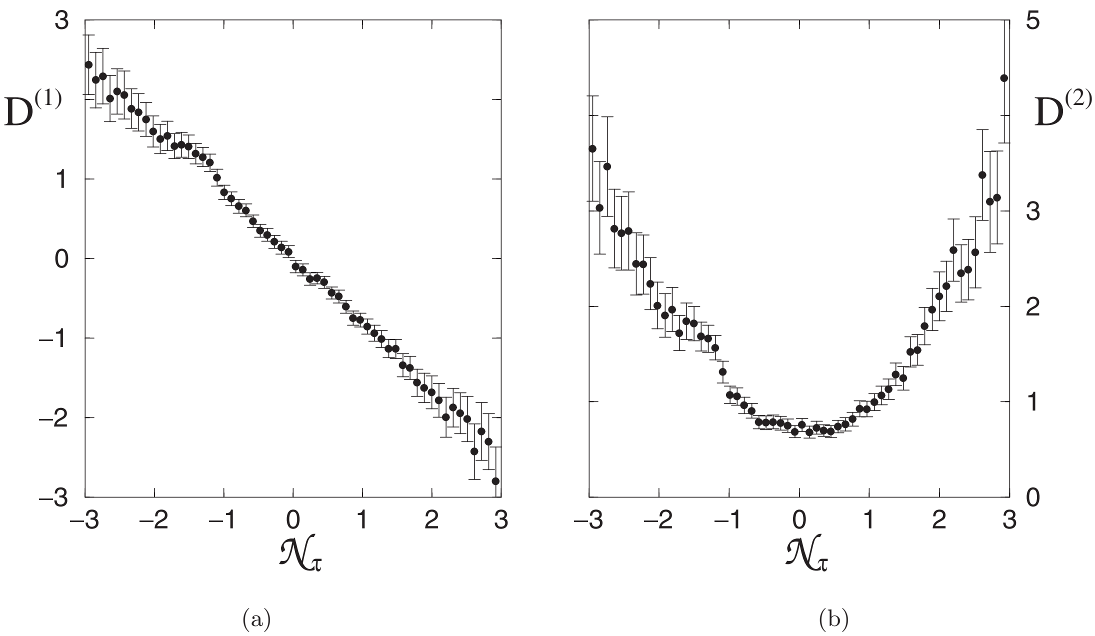
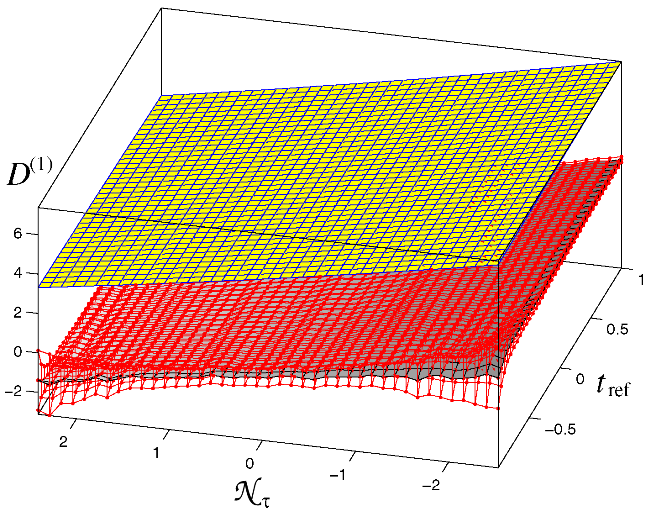
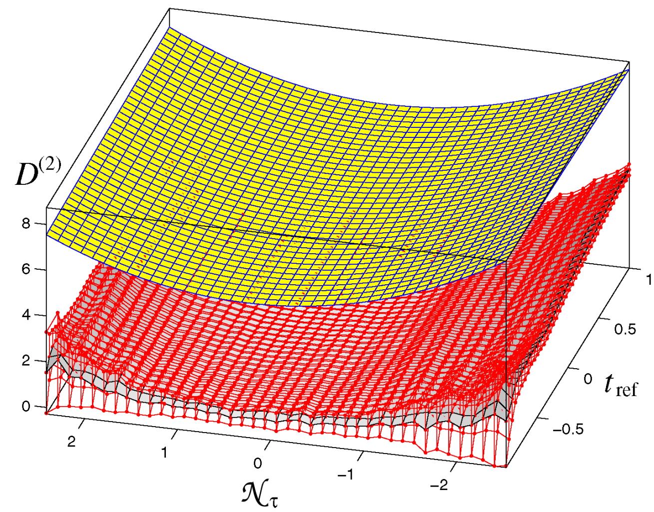


Fig. 4. The drift and diffusion coefficients, D_1 and D_2 in Eq. (1), of the daily index, obtained from linear fits of the corresponding conditional moments, beyond the Markov length (see Fig. 3). Here $t_{\text{ref}} = 90$ and $\mathcal{N}_\tau$ is given in the same units as the NAO index.



(a)



(b)

Fig. 5. Dependence of both drift and diffusion coefficients of the daily index on the time of reference t_{ref}. As one sees, for both coefficients this dependence is weak (see text). Here t_{ref} was normalized into the interval $[-1, 1]$, using values from 24 to 180.

For the monthly index one can also neglect the dependence on t_{ref} and the parameterizations of the corresponding KMC are

$$D^{(1)}_{\text{month}}(\mathcal{N}_\tau) = 0.2285 - 1.0659\mathcal{N}_\tau, \quad (5)$$

$$D^{(2)}_{\text{month}}(\mathcal{N}_\tau) = 1.6507 - 0.2428\mathcal{N}_\tau + 0.4954\mathcal{N}_\tau^2. \quad (6)$$

For both indices, the negative slope of the drift coefficient indicates the existence of a damping force. In addition, the quadratic term in both diffusion coefficient indicates possible intermittency phenomena in the evolution of the indices.

By integrating the associated Fokker–Planck equation [Risken, 1984] one tests the validity of our approach, which assumes from the very beginning that the process is Markovian, i.e. the conditional probability constrained only to the previous time-step equals the conditional probability constrained to any number of previous time-steps. For both, the daily and the monthly indices, PDF obtained by integration are in good agreement with PDF extracted directly from the data sets.

3. Reduction of Stochasticity

For both indices the diffusive term is significantly larger than the deterministic term, reflecting the stochasticity of the NAO indices. As we recently showed by a variationally optimized Markov analysis [Lind *et al.*, 2005], the high stochasticity in the monthly NAO index evolution can be reduced approximately by a factor of 3 when considering the two underlying pressure time-series, P_1 and P_2, as a two-dimensional coupled system. One starts by parameterizing the two coupled Langevin equations corresponding to the evolution of each pressure time series, namely

$$\frac{dP_1}{dt} = h_1 + g_{11}\eta_1(t) + g_{12}\eta_2(t), \quad (7)$$

$$\frac{dP_2}{dt} = h_2 + g_{21}\eta_1(t) + g_{22}\eta_2(t), \quad (8)$$

whose coefficients h_i and g_{ij} represent the deterministic and stochastic contributions, respectively, and η_1 and η_2 denote two independent δ-correlated Gaussian noise terms.

A general transformation of variables, $(P_1, P_2) \to (N_1, N_2)$, in Eqs. (7) and (8), leads to a new system with coefficients g' and h'. The main point in our procedure is to require that for one of the new equations, say the one in N_1, both stochastic terms become as small as possible when compared to the deterministic part. In other words, the dependence of N_1 on P_1 and P_2 is such that a functional F is minimized, namely

$$F = \frac{\|g'_{11}\|^2 + \|g'_{12}\|^2}{\|h'_1\|^2}, \quad (9)$$

where $\|f\|$ is the L_2-norm of f. Imposing that the norm in the denominator is some suitable constant [Lind *et al.*, 2005], one has without loss of generality, a variational problem with the

Lagrangian

$$L = \left(g'_{11}\right)^2 + \left(g'_{12}\right)^2 + \left[\left(h'_1\right)^2 - 1\right]. \tag{10}$$

Thus, solving numerically the Euler–Lagrange equation for this Lagrangian yields the "optimal" index N_1 for which stochasticity is as small as possible in the sense described above. This procedure can be generally applied to any system of coupled Langevin equations, in particular to the daily index, for which the Markov analysis is much more reliable than for the monthly index, due to its larger correlation length.

Comparing the parameterizations of the indices above [Eqs. (3) through (6)], one concludes that both the multiplicative and additive dynamical noises are larger for the monthly index. This can be seen from the diffusion coefficient $D^{(2)}$ which gives a measure of the dynamical noise, namely, the terms depending on $\mathcal{N}_\tau$ result from the multiplicative dynamical noise while the independent term indicates the amplitude of additive noise.

It should be noticed that the accuracy of KMC depends on the size of the data set and on the amplitude of an additional measurement noise. While for both indices the available data sets are small, the daily index time series (16801 data points) is about one order of magnitude longer than the monthly index (2135 data points). As for the measurement noise, indications of its presence can be found in Figs. 2 and 3. Namely, the tendency of the power spectrum to be constant at high frequencies for the daily index [Fig. 2(d)] and the divergence of the conditional moments when $\Delta t \to 0$ (Fig. 3). There is also a quantitative procedure which allows an estimation of the amplitude of the measurement noise [Siefert *et al.*, 2000] only by computing the second cumulant $K^{(2)} = M^{(2)} - (M^{(1)})^2$. However, due to the sparse data sets available, the cumulant yields very crude results.

4. Discussion and Conclusions

In this paper we have presented a Markov analysis to the common monthly NAO index comparing it with a similar NAO index, computed from daily pressure data. While, in both cases the dynamical noise and consequently the stochasticity is large, the daily index contains a smaller stochastic term than the monthly index. Consequently, the daily data are more reliable to study and characterize the NAO system than the usual monthly data.

To improve predictability of the daily index, we described a recently introduced variationally optimized Markov analysis, based on the minimization of a suitable functional, leading to a new index with less stochasticity. It should be of importance in forthcoming studies to analyze the time-series of the optimal indices. Furthermore, since the new indices are not simple differences between the pressures at both NAO poles, a new interpretation of the phases of the bipolar oscillation must be given in order to understand the physical meaning of such new indices.

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