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PHYSICA A

Simulating memory effects with discrete dynamical systems

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We propose using discrete dynamical systems to model non-Markovian processes. This implies a whole *hierarchy* of dynamical systems with dimensionality increasing proportional to the memory. Specific long-range memory effects are investigated for a quadratic, a cubic and a quartic map. Markovian processes transform the quadratic map into a normal form of the logistic equation; the Hénon map corresponds to non-Markovian "first-generation" memory effects. Higher memories imply new high-dimensional systems. Non-Markovian processes imply new "memory-routes" to chaos. For the three polynomial maps discussed here it is possible to define "critical memory lengths" above which the systems essentially lose memory.

There are several dynamical systems of interest which involve effects well approximated if one assumes them to occur at the same time as the cause responsible for them. It is not difficult, however, to imagine situations in which effects observed at a given time are actually consequences of causes which occurred earlier. In such cases the system acts as if it had a memory of earlier events. Examples of simplified models to deal with what might be called *memory effects* are difference-differential equations used to describe some physiological control systems [1] and the behavior of the electric-field amplitude of a plane-wave ring cavity containing a cell with a gaseous sample of two-level atoms [2]. The mathematical description of phenomena involving time-delays is usually done using difference-differential equations. Such equations are frequently regarded as defining "infinite dimensional" dynamical systems. Let $x(t)$ represent a quantity of interest, depending on a *continuous* time t. Difference-differential equations involve terms in $x(t-\tau)$, i.e. on the value of the variable at an earlier time τ. Infinite–dimensionality refers to the

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need of specifying initial conditions for the whole interval $0 \leq t \leq \tau$. The study of high-dimensional dynamical systems is presently a topic of very active research [3]. Apart from their utility as physical models, difference-differential equations have also been used as test-ground for extracting information using "embedding techniques", specially via computation of various correlations, dimensions, entropies, etc. Recent references include [4] from where further references might be obtained. A problem in dealing with difference-differential equations are spurious effects introduced via mandatory discretizations needed to integrate them. In other words, the advantage of simulating high-dimensionality with difference-differential equations might be impaired by unwanted "integrator effects".

This paper proposes using *discrete maps* as a convenient framework to study memory effects. The *rationale* for our proposal is basically the same used to fruitfully approximate systems of ordinary differential equations by discrete maps: (i) it is far easier to iterate a map than integrate differential equations, (ii) numerical uncertainties involved in iterations are much smaller and "controlable" than those involved in integrations and, the essential characteristic, (iii) it works. All previous works involving discrete dynamical systems that this author is aware of are based on what in the framework being proposed here can be interpreted as Markov processes modeled via equations of the generic form $x_{t+1} = f(x_t; p)$, p representing collectively one or several parameters. This paper reports results obtained by considering for the first time *long-range* memory processes simulated via *discrete* dynamical equations. In other words, we propose simulating full non-Markovian processes via general models written generically as

$$x_{t+1} = f(x_t, x_{t-1}, \ldots, x_{t-(N-1)}; p) , \tag{1}$$

where N is the *dimension* of the system, $N-1$ is the *order* of the memory and f the model. As a specific example, we present results obtained by perturbing the quadratic map $x_{t+1} = a - x_t^2$ via reinjection of the *field* $x_{t-\tau}$ (measured at earlier times $\tau = 1, 2, 3, \ldots$) with a feedback amplitude b:

$$x_{t+1} = a - x_t^2 + b x_{t-\tau} . \tag{2}$$

The whole hierarchy of equations implied by eqs. (1) and (2) and their interpretation as resulting from memory effects are the main results of this paper. In the remainder of the paper we present properties/consequences of the memory effects incorporated in eq. (2). A preliminary report about memory effects simulated via discrete dynamical systems is submitted elsewhere [5].

For $b = 0$ (zero feedback amplitude), the dynamical behavior of eq. (2) is textbook knowledge: bounded (finite) attractors exist only for $a_0 \equiv -1/4 \leq$

$a \leq a_c \equiv 2.0$. Between $a_0 \leq a < a_1 \equiv 3/4$ it has stable fixed points; stable period-2 cycles exist for $a_1 \leq a < a_2 \equiv 5/4$; period-4 up to $a \simeq 1.3681\ldots$, bifurcating further until $a_\infty \simeq 1.40115\ldots$ where *chaos* begins. For $a > a_\infty$ there are intervals of periodicity, for example, a period-3 window starts at $a_T = 7/4$. It is possible to prove a number of results for eq. (2): (i) it is always a diffeomorphism for $b \neq 0$; (ii) its Jacobian is always constant and given by b multiplied by the "parity" $(-1)^\tau$; (iii) the equations defining fixed points do not depend on τ, their non-trivial term being $x^2 - (b-1)x - a = 0$; (iv) for any τ the *stability* of fixed points is determined from the polynomial

$$\lambda^{\tau+1} + 2x\lambda^\tau - b = 0\,; \tag{3}$$

(v) there is a similar full hierarchy of τ-dependent equations defining the stability of all higher periods. While *domains* of periodicity are still determined from exactly the same equations as for the Hénon map, their *range of stability* is now a highly interleaved "manifold-like" structure induced by increasingly high-degree polynomials in τ.

The case $b \neq 0$ with $\tau = 0$ represents a trivial change of the quadratic normal form to the familiar *logistic equation*. Its dynamics is isomorphic to that for $b = 0$ and therefore will not be further discussed. Note however the new point of view associated to the transformation between the quadratic and the logistic maps.

Case $\tau = 1$. Defining $y_t \equiv x_{t-1}$, eq. (2) yields the 2D system

$$x_{t+1} = a - x_t^2 + by_t\,, \qquad y_{t+1} = x_t\,. \tag{4}$$

This equation is a normal form of the Hénon map, a very well studied discrete map for which voluminous results exist in the literature [6]. From the well known properties [6] of the Hénon map it is immediately clear that by properly controlling the feedback-amplitude b one is able to generate all the rich phenomena typical of the Hénon map, i.e. attractors of any periodicity as well as chaotic behaviors. So far, virtually all studies of the Hénon map have been done by considering the dynamics as a varies along lines of constant b. The present formulation of the problem shows how to interpret results obtained by varying b along lines of constant a. The Hénon map is a diffeomorphism for all $b \neq 0$ and has a constant Jacobian: $-b$. Therefore, fixing a and varying the feedback amplitude b corresponds to continuously varying the Jacobian. To see memory effects refer please to fig. 1. This figure shows isoperiodic diagrams on the space of parameters of eq. (4) obtained as described in ref. [7] for the first 4 values of τ. (Ref. [7a] contains more than 20 color isoperiodic diagrams

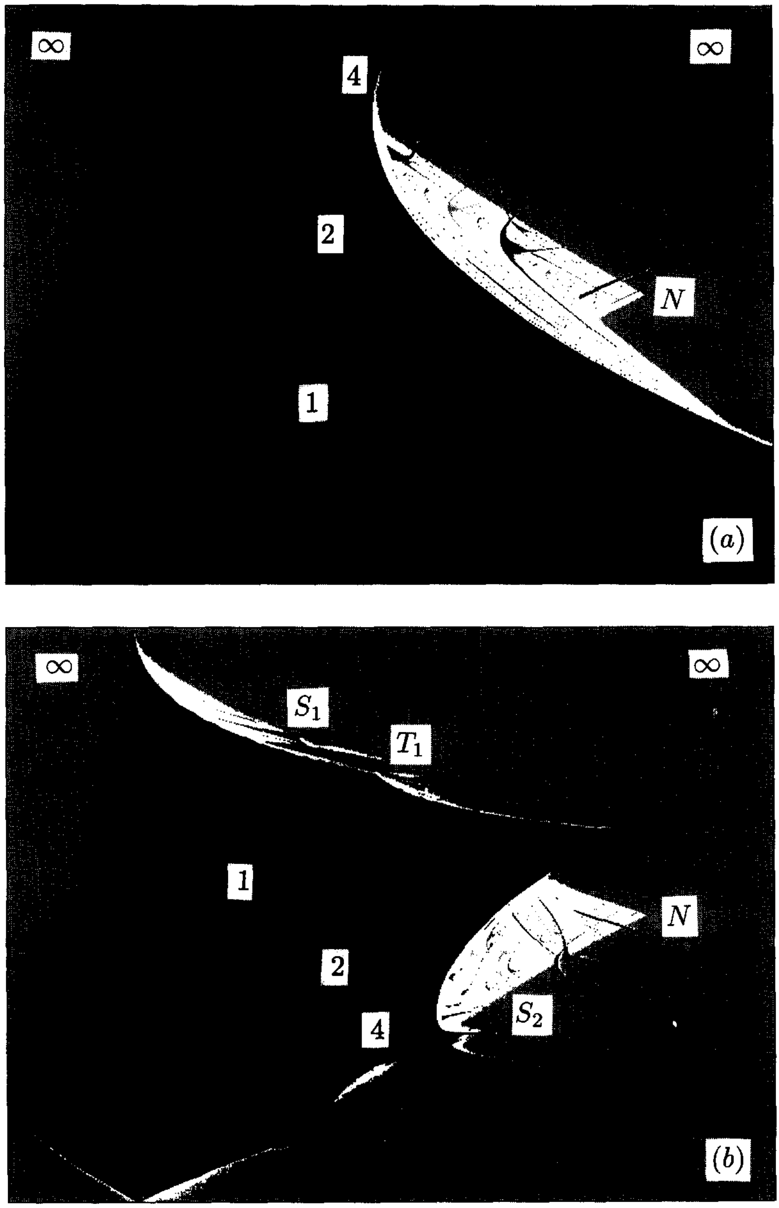
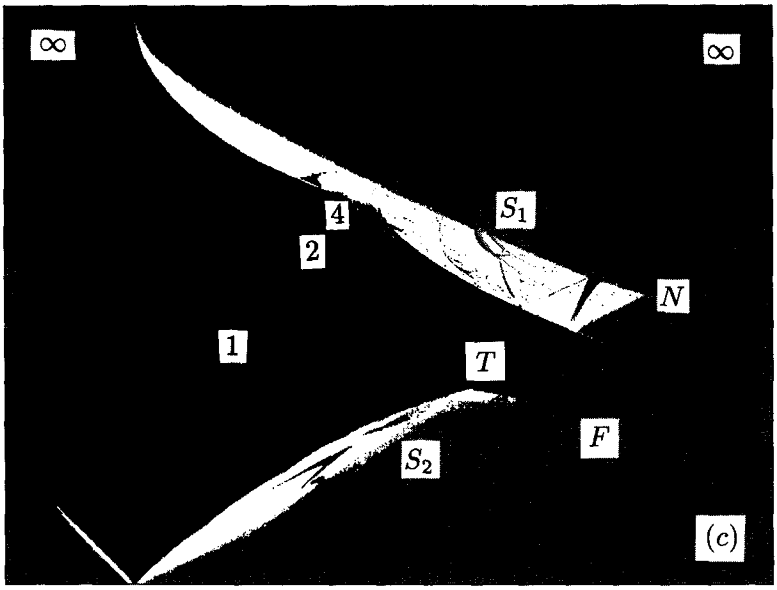


Fig. 1. Isoperiodic diagrams showing memory effects in the quadratic map, eq. (2), as a function of the parameter a (horizontal axis), the feedback amplitude b (vertical axis) and time-delay τ. In this figure the coordinates of bottom-left corners are always $(a, b) = (-0.5, -1.0)$. Upper-right coordinates are (2.5, 1.0). Both scales are linear. Numbers represent periods found by *following orbits* [7] from *diagonal zeros*, i.e. $x_0 = y_0 = 0.0$. Similar shading indicates identical periodicity. White regions represent domains of chaotic behaviour. N refers to the 'nose' at $a = 2$ along the invariant Markov line $b = 0$. The gray shading containing the symbol ∞ refers to unbounded



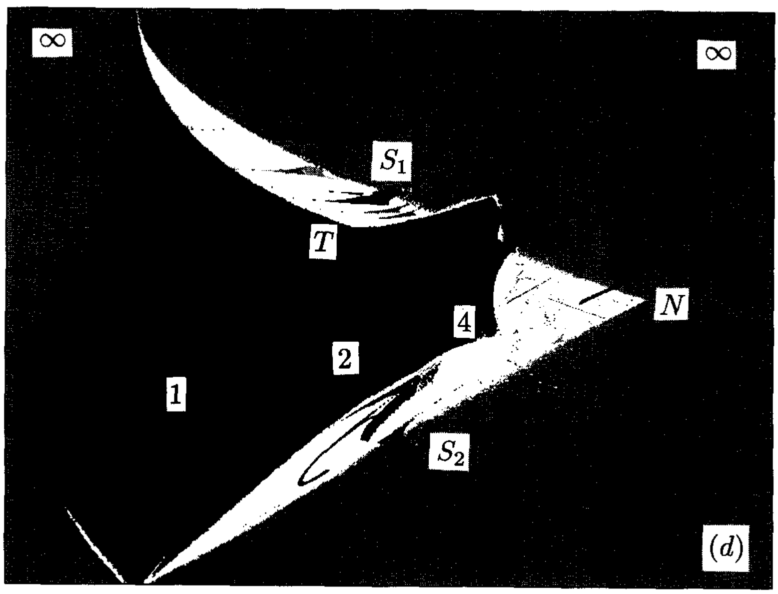


Fig. 1 (cont.).

attractors. In some figures, T_i refers to *triplex points* and S_i to *shrimps* (see text). Although hard to see in the figures, adjacent to the period-4 regions there are always domains where full doubling cascades develop. (a) $\tau = 1$: the 2D Hénon map; (b) $\tau = 2$: a 3D map; (c) $\tau = 3$: a 4D map; F represents a *fractal domain* in the parameter space (see text); (d) $\tau = 4$: a 5D map.

displaying in details the rich topology of the shrimps embedded in the chaotic domains.) Each shading represents domains of points (a, b) for which orbits of $x_0 = y_0 = 0.0$ are found to have the same periodicity. The shaded regions containing the symbols ∞ indicate parameter values for which there are no bounded attractors. Black regions containing the number 1 refer to domains of fixed points, i.e. to parameters for which eq. (4) has period-1 orbits. The shaded regions containing the number 2 indicate period-2, etc. Adjacent to the period-4 regions there are complete 1×2^n cascades which are, however, hard to see on the scale of the figure. White regions denote the *Via Caotica*, i.e. parameters corresponding to chaotic orbits. All periods up to 20 have been represented in the figure. As can be recognized by the strong compression of the domains as the period increases, it is virtually impossible to discern periods higher than about 15 on the scale used. This means that white domains contain embedded in them many very thin islands of higher periodicity that can not be properly represented in the scale of the figures (see however ref. [7]). The same shading convention was used for all figures in this paper.

Due to a known conjugacy [6] one only needs to consider the dynamics of eq. (4) within the strip $|b| \leq 1$. The map is orientation-preserving (reversing) below (above) the invariant $b = 0$ quadratic-map Markov line passing through the "nose" N at $a = 2$, $b = 0$. Memory effects can be seen by considering the dynamics along lines of constant a in fig. 1a. For $a < 0$ there are most of the times no bounded attractors, the exception being small domains of fixed-points. For $0 < a < 0.72$ ($a = 0.72$ is tangent to the period-4 domain) one finds either a fixed-point or period-2. The orientation-preserving domain contains virtually only fixed-point stable behavior. Above $a = 0.72$ the dynamical behavior observed by varying b becomes very rich quickly, specially for $b > 0$. There are two real fixed points for $a > -(b-1)^2/4$. From eq. (3) it follows that for $-(b-1)^2/4 < a < 3(b-1)^2/4$ one of them is a saddle while the other is *attracting*. These curves are exactly the two continuous parabolic borders delimiting the stable period-1 domain in fig. 1a. Similar analytical results might be obtained for the limits of the stable period-2 domain. For higher periods one can easily obtain equations defining the borders but to solve them is a non-trivial task. Ref. [7] discusses at length the rich dynamical behaviors existent inside the white chaotic domains. In such domains one finds many regularities, specially families of isoperiodic "shrimps" aligned along specific directions [7]. We merely indicate by S_i some regions where shrimps concentrate and refer to ref. [7] for a detailed description of the complex phenomena inside chaotic domains. These phenomena are not relevant for our purposes here (see however ref. [5]).

Case $\tau = 2$. Defining $z_t \equiv x_{t-2}$, eq. (2) yields the following 3D system:

$$x_{t+1} = a - x_t^2 + bz_t\,, \qquad y_{t+1} = x_t\,, \qquad z_{t+1} = y_t\,. \tag{5}$$

The Jacobian is now b. Fig. 1b shows how the two-generations memory affects the dynamics. The most evident effect is that fixed-point behaviour is now restricted to a much smaller range of parameters. In addition, in sharp contrast with what is commonly observed in *standard* discrete models of dynamical systems, the familiar steady bifurcation cascade 2^n appears interrupted in various parameter domains. Specially interesting regions are those converging to the several "triplex-points" T_i, borders between *three* domains of different stable motions. Near such points, small perturbations (noise) are able to induce complicated multistable and hysteretic behaviors. Note that radical changes happen at both ends of the $a = 0$ line where for any $\tau > 1$ one finds "cusp-like" behaviour. Discontinuous changes in derivatives while moving along borders can also be seen in other regions of the parameter space, notably at the T_i. These characteristics imply the existence of many "memory-routes" to chaos different from those known.

Cases $\tau = 3$ *and* $\tau = 4$. Fig. 1c and 1d show memory effects for the 4D and 5D systems, respectively, obtained analogously to those in eqs. (4) and (5). Figs. 1a and 1b, respectively 1c and 1d look "similar", reflecting the common parity of their Jacobian. Further, the pictures seem to loosely "rotate" around the quadratic-map line $b = 0$ as τ increases. Note however the conspicuous "shoulder" at the left bottom corner in all pictures. Indicated by F in fig. 1c is a fractal structure in the parameter space. Such structures reflect the coexistence of more than one bounded attractor: for parameters in this range, plots on a rough scale of the basin of attraction appear to divide the space of variables into two domains corresponding to a bounded attractor and to the attractor at infinity. However, upon closer examination it is possible to uncover at least one other much thinner filament-like basin "embedded" in a very complex and beautiful way in the basin of the bounded attractor. Adiabatic parameter changes while generating isoperiodic diagrams from a single fixed initial condition as done here [7] are equivalent to traversing across the filaments. This exposes the complex intertwining of the basins by causing the system to switch between different attractors thereby generating the fractal structures in the parameter space. Such fractal structures in the parameter space do not seem to have been observed before. As for $\tau = 2$ one also finds in figs. 1c and 1d sudden interruptions of period-doubling cascades with the system "jumping" directly into chaos. Although the precise conditions for the occurrence of this phenomenon are not yet clear, it is interesting to observe that such a situation has already been observed experimentally [8]. This same type of phenomenon might perhaps explain some other "interrupted" routes to chaos measured near tangencies in glow-discharge plasmas [9].

By considering isoperiodic diagrams for $\tau = 11, 21, 31$ and 41 (which have all the same parity as the Hénon map) one recognizes that the domain of bounded trajectories is strongly reduced and displays relatively small changes as τ increases. In fact after about $\tau = 11$ the system can be considered to have essentially "lost the memory". From this it seems fair to say that there is a characteristic *memory threshold* above which the system has no memory. What happens for much longer-range memories? Fig. 2 shows an isoperiodic diagram for a feedback process with $\tau = 101$, chosen to have again the same parity as the Hénon map. This corresponds to a 102-dimensional system. Perhaps the most striking feature in this picture is the relative symmetry with respect to the $b = 0$ Markovian line, the border line across which the Jacobian changes sign. A relative "insensitivity" to changes in the sign of the Jacobian is of interest because of the different roles frequently attributed to the orientation-preserving or reversing maps, specially in higher-codimensional dynamical systems. The relative insensitivity to the sign of the Jacobian can be already observed at

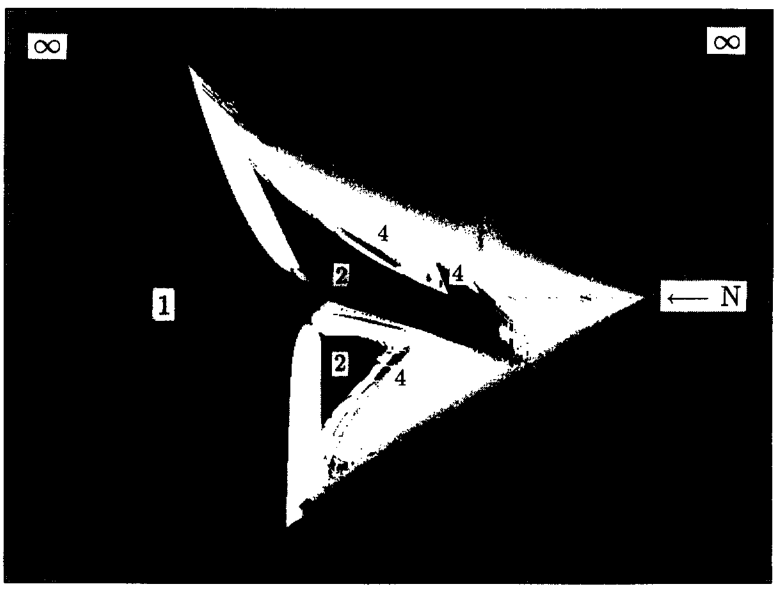


Fig. 2. Isoperiodic diagram showing long-range memory effects corresponding to $\tau = 101$ (same parity as in figs. 1a and 1c). Scale and symbols as in fig. 1. Note the relative symmetry with respect to the $b = 0$ Markov line when compared with the short-range Hénon process shown in figs. 1a and that in fig. 1c. Domains of bounded attractors are much reduced and chaos takes over. Scales and conventions as in fig. 1. The "4" inside *Via Caotica* refers to the periodicity of the adjacent domains.

much lower values of τ. By drawing isoperiodic diagrams for increasingly higher values of τ one obtains figures which are more and more symmetric with respect to the $b = 0$ Markov line. This numerical evidence appears to indicate that the $\tau = \infty$ limit might perhaps correspond to a situation where orientation-preserving and orientation-reversing domains in the parameter space have identical topologic characteristics.

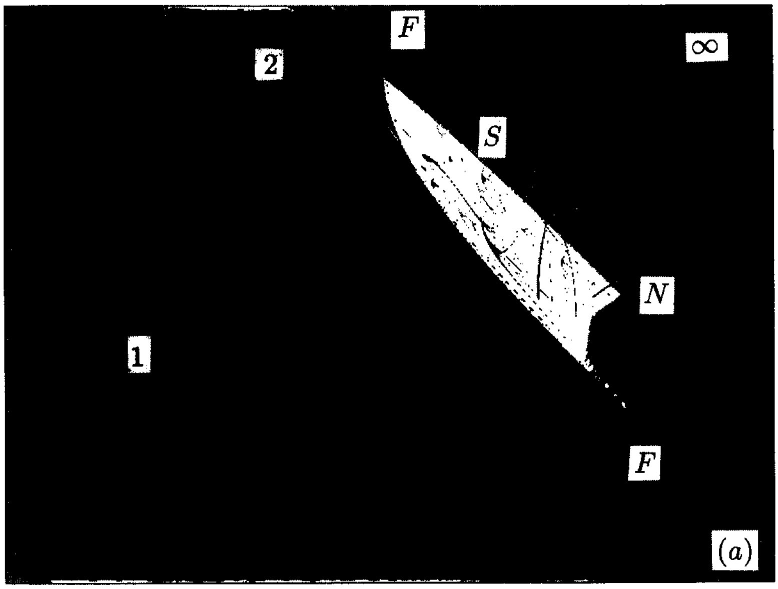
By comparing figs. 1a and 2 it is possible to see that the net effect of the memory is roughly to exchange domains of stable periodic behaviors into domains of chaos. In particular, memory effects appear to conspire to quickly wash out high-period orbits. A measure of this effect can be obtained by comparing the sensible reduction of the period-1 domains between figs. 1a and 2. Note also that the period-4 that appears always adjacent to the period-2 domain is very much reduced. To observe higher periodic orbits requires zooming into specific zones, shrimp zones [7], of the space of parameters. Fig. 1 clearly shows multiple triplex points. As τ increases these points move towards the $b = 0$ line. In particular there is a very stable triplex point, common border of period-1, period-2 and chaos, located slightly below the $b = 0$ line which tends to it as τ increases.

A typical phenomenon observed in dynamical systems is the coexistence of more than one *stable* attractor for a given set of parameters. The initial conditions are the important factors determining to which stable attractor the system will eventually go. By considering isoperiodic diagrams obtained for several different initial conditions one sees that as the absolute value of x_0 increases there is a strong reduction of the cobasin of bounded attractors. It is also possible to observe domains where the border between parameters corresponding to diverging and non-diverging orbits is very well approximated by line segments. As in figs. 1 above, the region close to the triplex point denoted by T in fig. 1d is found to be stable not only with respect to changes of τ but also with respect to changes of initial conditions. There is another triplex point at about the same vertical position as T and roughly between the nose and T. However it is not as stable as T. It would be interesting to investigate these points and the dynamics around them more closely.

After considering memory effects on the quadratic map it is natural to look for the same effects in other systems. I considered two further dynamical systems which are 'very close' to the Hénon map. These systems are inspired by an early work of Holmes [10] who used what might be perhaps called in the present context 'cubic variants of the Hénon map' to model Duffing equations. Fig. 3 shows isoperiodic diagrams for $\tau = 1$ and 2 for the system

$$x_{t+1} = x_t(a - x_t^2) + bx_{t-\tau} \,. \tag{6}$$

By comparing figs. 1a and 3a one sees many characteristics of the space of



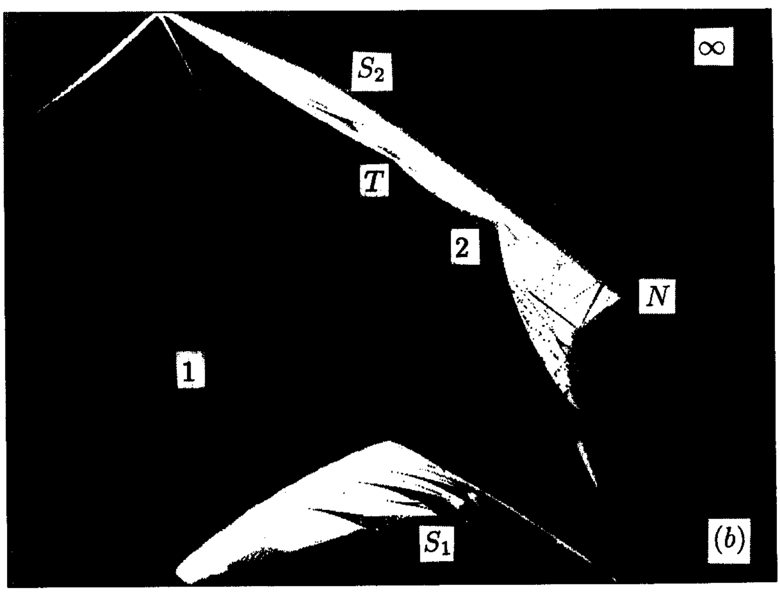


Fig. 3. Memory effects for the cubic map, eq. (6), as a function of a (horizontal axis), b (vertical axis) and τ. In both figures the coordinates of the bottom-left corners are $(-1.0, -1.0)$ while upper-right coordinates are $(4.0, 1.0)$. Scales are linear and orbits were started at $x_0 = 0.5$. Further symbols are as defined in fig. 1. (a) $\tau = 1$; (b) $\tau = 2$.

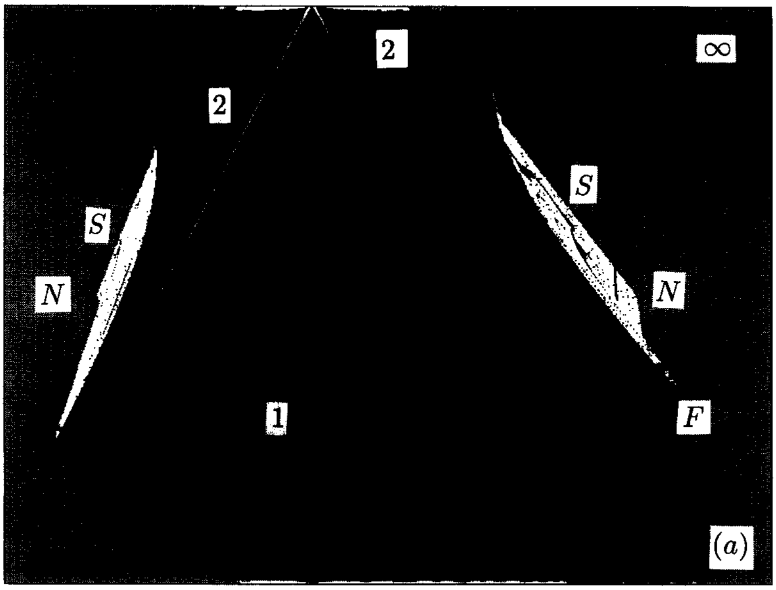
parameters seem to 'survive' the change from quadratic to cubic dynamics. As for the Hénon map [7], the parameter space of eq. (6) also has fractal domains. The broad period-1 domains in figs. 3a and 3b are apparently divided in two pieces by a white line segment. In fact, this line is just a consequence of the number of preiterates being not enough to overcome the quite slow convergence along it. A few test runs considering much longer transients showed that the white regions are in fact period-1 domains with very slow convergence. All other wide white regions (indicating regions of chaos) showed no change when increasing the number of preiterates discarded. Similarly to what is observed for the Hénon map [7], regions of predominant chaotic behavior are found to contain embedded in them several islands of isoperiodic dynamics. Triplex points are also present. Again, as τ increases, the domains of higher periodic behavior get smaller and smaller. An interesting fact is that the dynamics close to the $b = 0$ Markov line is quite stable against changes of τ. Detailed isoperiodic diagrams of families of cubic maps have been recently discussed in ref. [11]. This reference contains several color diagrams displaying the rich way in which islands of higher periodicity appear embedded in the wide chaotic seas.

discrete model of the Duffing equation? The answer for $\tau = 1$ and 2 can be seen in fig. 4 which displays isoperiodic diagrams obtained by considering the dynamics of the quartic equation

$$x_{t+1} = x_t(a - x_t^3) + bx_{t-\tau} \,. \tag{7}$$

As one sees from fig. 4a, there is a relative symmetry of the domains of high periodicity with respect to the vertical line $a = 0$. The "nose" is still present and appears now on both sides of the $b = 0$ Markov line. It is still possible to observe shrimps and fractal structures but they disappear fast as τ increases. By comparing figs. 1a, 3a and 4a it is possible to recognize some apparently "recurrent" characteristics of the simpler $\tau = 1$ two-dimensional systems. By considering the dynamics along constant a or b lines one sees that memory effects imply many possibilities of reaching chaos via sequences of bifurcations. A quick glance at the figures presented here is enough to convince oneself that the classification of even a few of the abundant and complicated routes to chaos is not a trivial task. A careful investigation of the "inferno" of possible behaviors contained in the space of parameters is no doubt of great interest. It is also interesting to observe that the cartographic information contained in isoperiodic diagrams precisely shows how to change parameters in order to jump between *stable* attractors, i.e. to *control* the dynamics of the system.

In conclusion, we proposed using discrete dynamical systems as models of long-range memory effects. The specific examples discussed were primarily intended to show how memory effects can be incorporated and described and



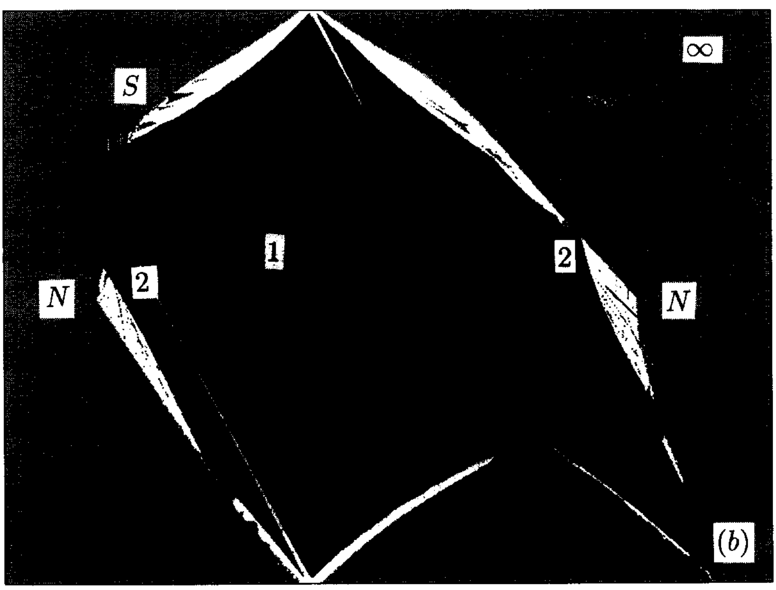


Fig. 4. Memory effects for the quartic map, eq. (7), as a function of a (horizontal axis), b (vertical axis) and τ. Coordinates of bottom-left corners are $(a, b) = (-1.0, 1.0)$; upper-right corners are at $(4.0, 4.0)$. Scales are linear and $x_0 = 0.5$. Further symbols are as defined in fig. 1. (a) $\tau = 1$; (b) $\tau = 2$.

to show some of their main consequences. The first example (eq. (2)) shows how to "unify" familiar models as the quadratic map, the logistic equation, the Hénon map, and a large family of other systems into a whole hierarchy of dynamical systems of increasing dimensionality. The interpretation proposed here implies the Hénon map to be in some sense "very close" to the quadratic map. The other examples were intended to show what happens when memory effects are taken into account in models of differential equations. The present work can be also regarded as presenting heuristic evidence that there is a connection between other families of high-dimensional systems and corresponding low-dimensional "building blocks" via memory effects. From the analysis of the isoperiodic diagrams presented here along with several others generated for different τ and initial conditions one sees that memory effects for the particular families of polynomial systems discussed in this paper produce basically similar changes on the parameter space. The 'generic characteristics' observed to recur in the systems discussed here are: (i) strong reduction of islands of higher periodicity as τ increases; (ii) appearance of triplex points; (iii) fractal structures between domains corresponding to bounded (finite) attractors; (iv) a relatively high symmetry between orientation-preserving and orientation-reversing domains as τ increases. While the models discussed in this paper certainly correspond to familiar dynamics observed in many fields of interest, there is no reason to expect generic memory effects to appear only via the quite simple expressions presently discussed. On the contrary. Therefore it seems reasonable to conjecture the exploitation of memory effects via discrete dynamical systems, either as a model of dynamics or as a tool to understand effects/characteristics of high-dimensions/codimensions, to be able to generate much interesting results. A particularly fruitful example is the family $x_{t+1} = a - x_t^2 - \tau$, which will be discussed elsewhere.

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