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Exploring collective behaviors with a multi-attractor quartic map

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Abstract

We simulate a 2D coupled map lattice formed by individual units consisting of a multi-attractor quartic map. We show that the interesting recently discovered *non-trivial collective behaviors* (where macroscopic quantities show well-defined, usually regular, temporal evolution in spite of the presence of local disorder in space and time) also exist, over wide parameter domains, in the presence of local periodic order, in systems having more realistic units allowing coexistence of more than one attractor. © 1998 Elsevier Science B.V. All rights reserved.

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1. Introduction

Since the last decade different systems composed of coupled map lattices (CML) have been studied as simple toy models used to unveil basic properties of spatially extended nonlinear dynamical systems. Conceived as far from equilibrium systems with many degrees of freedom, such systems can also be used to construct a statistical mechanics for irreversible process.

In the beginning of the decade, studying systems of locally coupled logistic maps, $x_{t+1} = \lambda x_t(1 - x_t)$, on spatial dimensions greater than one, Chaté and Manneville [1–4] found a characteristic temporal dynamics on spatial averages over the system emerging out of the local chaos, the so-called *non-trivial collective behavior* (NTCB). They found that such behavior happens when the single logistic map parameter λ is set to any value beyond the chaotic onset, that is for $\lambda > \lambda_c$. When $\lambda < \lambda_c$ the whole system synchronizes to the local periodic behavior. So, their interesting finding was

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that the basic ingredient to produce NTCB is local chaos competing with diffusive coupling.

Most works found in the literature [5] use the well-known logistic map to describe the local dynamics. A characteristic feature of the logistic map is the presence of, at most, just a single attractor at finite distances. In this respect, the logistic map is rather unique: modulo simple changes, it is perhaps the *only* map to consistently display this unique feature. Over rather extended parameter domains, generic maps usually display coexistence of more than one attractor at finite distance.

On a CML composed of such units the effect of the local coupling is to average the variable (field) over the neighborhood producing a new value inside the basin of attraction of the very same attractor. The motivation of this work is to investigate a CML possessing two non-infinite attractors. In order to do that we use for the local dynamics the one-dimensional quartic map $x_{t+1} = (x_t^2 - a)^2 - b$, introduced by one of us in 1992 [6]. This map might be thought as the second iterate to the quadratic (logistic) map in which the parameter a was replaced by an arbitrary parameter b. Its characteristics have been discussed at length in Ref. [7].

2. Model and simulations

Starting from random initial conditions we simulate a system composed of N individual units whose dynamics at the site i follows the quartic map [6,7]

$$y_i^{t+1} = [(x_i^t)^2 - a]^2 - b\,,$$

where x_i^t is a real variable representing some quantity of interest measured at time t; a and b are real parameters. After each generation t the value of y_i^t is averaged over the first neighbors,

$$x_i^t = (1 - \varepsilon) y_i^t + \frac{\varepsilon}{2D} \sum_{j=1}^{n} y_j^t\,,$$

where $D = 2$ is the spatial dimension of the system and $n = 5$ is the number of first nearest neighbors of site i, including the site i itself. The geometry used for the simulation presented here is that of a square lattice of sizes up to $N = 512 \times 512$ units.

We first explored the system scanning the space of parameters $a \times b$ in the interval $\{-1, 2\} \times \{-1, 2\}$ dividing it with a resolution of 512×512. For each point (a, b) of parameters we iterated a small lattice of 64×64 sites, up to one thousand times departing from initial conditions randomly chosen from the basins of attraction of the uncoupled quartic map. We used periodic boundary conditions. From such experiment we could easily discriminate the set of parameters for which the lattice rapidly synchronizes from those which present either a long transient or a non-trivial collective behavior.

3. Results and discussion

In Fig. 1 we show the parameter sets obtained by painting the parameter space according to the magnitude of the Lyapunov exponents of the local map. Black domains show sets of parameters where the coupled system does not synchronize up to 1000 iterations *and* the local uncoupled map is periodic, the white color represents regions where the system does not synchronize *and* the local uncoupled map is chaotic, the dark gray shows the regions where the coupled system synchronizes to the local uncoupled map and the pale gray color shows parameters for which orbits are attracted to infinity.

The striking novel result being reported here are the black domains in Fig. 1. *In the black domains we find NTCB without chaos on the local uncoupled oscillator*. This absence of synchronization of the variable over the space could well be just a very long transient [3]. To clear out this possibility we explored in detail the point $a = 0.35$, $b = 1.35$ in the black region. For these parameters, the local uncoupled map has two fixed points, $x_1 = -1.227591$, $x_2 = -0.011399$ and negative Lyapunov exponents $L_1 = L_2 = -1.200$. After a small transient the coupled system presents a period two NTCB with the instantaneous average over the variable jumping consecutively from inside the interval $\{0.125144,\ 0.815711\}$ to the interval $\{-1.349951, -1.245386\}$. Fig. 2 shows two snapshots, recording the typical appearance of the lattice in each of these intervals. We have performed 2.5×10^5 iterations with large lattices of 512×512 sites and up to 10^6 interactions with smaller lattices of 128×128 sites without observing synchronization on the system. Thus, it seems that the novel phenomenon is robust over relatively long periods of time.

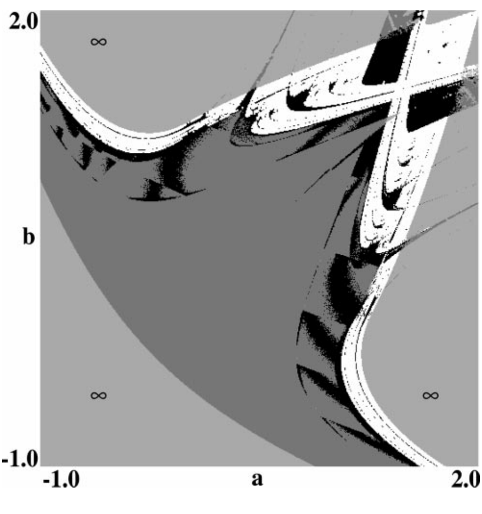

Fig. 1. Characteristic behaviors as seen in parameter space. Black: region where the single quartic map is periodic *and* the coupled system does not synchronize. White: the single map is chaotic and the coupled system does not synchronize. Dark gray: coupled system synchronizes to the local uncoupled map. Light gray, i.e. the domains containing the symbol "∞": parameters having orbits attracted to infinity.

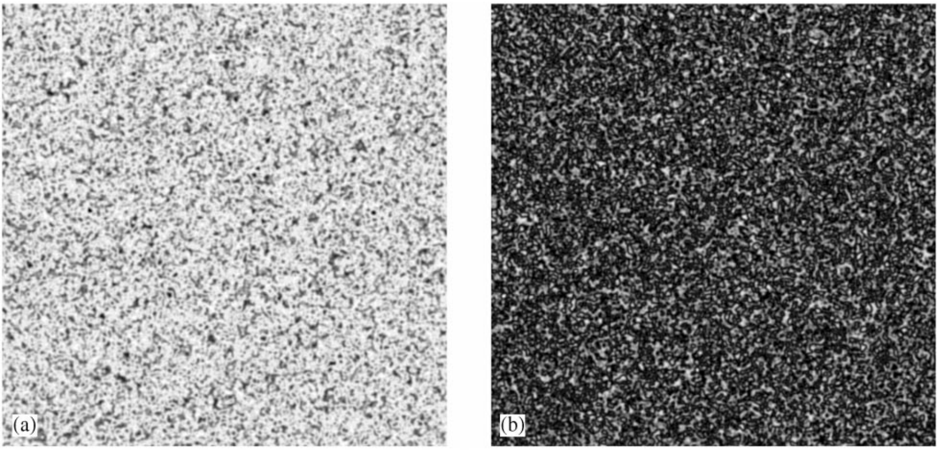

Fig. 2. Two consecutive snapshots of the spatial variable (x) showing a period two NTCB for $a=0.35$ and $b=1.35$. The gray level scale on the left, (a), is in the interval $\{0.125144, 0.815711\}$ while that on the right, (b), is in the interval $\{-1.349951, -1.245386\}$.

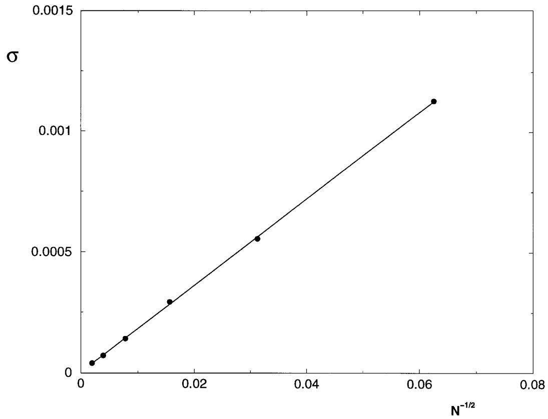

Fig. 3. Time fluctuation of the mean deviation of the lattice spatial average for different lattice sizes N. See text.

In order to confirm the stability of the NTCB for the point $(a,b)=(0.35,1.35)$ in parameter space we studied the dependence of the time fluctuation of the spatial average of the field variable with the lattice size during the same time interval. These results are shown in Fig. 3. For large systems this average is expected to become more and more defined [1,3]. In fact, for increasing lattice sizes the fluctuation decreases with the $\sqrt{N}$ (see Fig. 3) and the NTCB becomes better and better defined. Another test of the robustness of the NTCB is to verify whether or not it is stable for changes of the parameters. To this end, we performed several additional simulations of the system inside a region of radius 0.01 centered around $(a,b)=(0.35,1.35)$ in the parameter space. We have found the same period two NTCB with small changes on the spatial averages showing that, indeed, the novel phenomenon is also robust to changes in parameters.

From the original work of Chaté and Manneville [1] it was already clear that NTCB existed for lattices of logistic maps driven inside the so-called periodicity windows which abound when $\lambda > \lambda_c$. A similar result has also been remarked by one of us in a collaboration with Chaté and Manneville [8] when exploring the robusticity of the NTCB for the coupled continuous-time Rössler system. In both cases, NTCB for periodic parameters emerges on a small window surrounded by chaos in parameter space. In the situation being presently reported, however, NTCB emerges on the boarding region between periodic and chaotic behavior.

From these results we conclude that sensibility to initial conditions (SIC) on the local attractor is not a necessary condition to see NTCB: we found NTCB even when the local dynamics is *periodic* over rather extended domains in parameter space. Nevertheless, the existence of spatial dispersion clearly indicates that the diffusive coupling, which forces the system to synchronize, is counterbalanced by some kind of SIC. This means that knowledge of dynamics of the local attractors does not imply knowledge of the global attractor which emerges for the coupled system. We may try to generalize summing up the simulations presented here and the previous results: for NTCB to exist on a periodic local base we need either (i) a phase space containing many different basins of attraction, i.e. allowing *coexistence of more than one attractor* at finite distances, or (ii) a small window of periodicity surrounded by chaos. The first of these two conditions is the main message of the present paper.

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