Recurrent inflation and collapse in horizontally shaken granular materials

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We report a striking effect observed experimentally in several granular materials when shaken horizontally: The material displays a recurrent alternation between a slow *inflation phase*, characterized by an increase in its volume, and a fast *collapse phase*, when the volume abruptly returns to its original value. The frequency of such phase alternations is totally decoupled from the frequency of the external drive. We argue that the inflation and collapse alternation arises from an interplay between the mechanical stability of the material and Reynolds dilatancy due to convective motion.

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I. INTRODUCTION

Many key results about the dynamics of complex materials were discovered during the last two decades by studying what happens when containers filled with granular material are shaken. Among the most enticing problems investigated extensively we mention the perennial quest for understanding subtle and elusive aspects of granular convection as well as the efforts to characterize transitions between distinct granular states like, e.g., solid and fluid [1–10].

Under *vertical* shaking, granular dynamics has been now extensively studied experimentally, analytically, and by means of computer simulations, so that there is already a fair understanding of the basic phenomena involved [1–10]. In contrast, for *horizontal* shaking (or for combined horizontal and vertical shaking), we are still far from a similar level of understanding although there are already a number of results available [11–24]. The aim of this paper is to report a flabbergasting effect that we observed experimentally in several granular materials when shaken horizontally under rather general conditions.

The remarkable effect reported here is a recurrent swelling of the grains, in which they undergo a cyclic alternation between two phases: a slow *inflation phase* characterized by an increase of its volume, followed by a relatively fast *collapse phase*, a quick relaxation of the material, when it returns abruptly to its original volume. These periodic phase oscillations are easily observable under cyclic horizontal shaking of the container, $x(t) = A\cos(2\pi f t)$, for specific ranges of amplitude A and frequency f and for certain intervals of the filling level. Below we argue that the height fluctuations, the intensity of the convection flow, and the energy dissipation rate in the flowing material are closely related effects.

II. EXPERIMENTAL SETUP

Swelling oscillations are not difficult to observe. We have measured them for quite different materials (quartz sand, aluminium-oxide powder, ferro-oxide powder), for different grain sizes (20 μm $\leqslant r \leqslant$ 200 μm), in different containers (length ranging from 6 to 10 cm), and for shaking amplitudes A in the range 0.1 cm $\leqslant A \leqslant$ 0.3 cm. In all cases we were able to observe the swelling effect in certain ranges of the parameters, filling height, and frequency. As representative of the results typically observed, here we report on experiments done with quartz sand with an average grain size of about 100 μm.

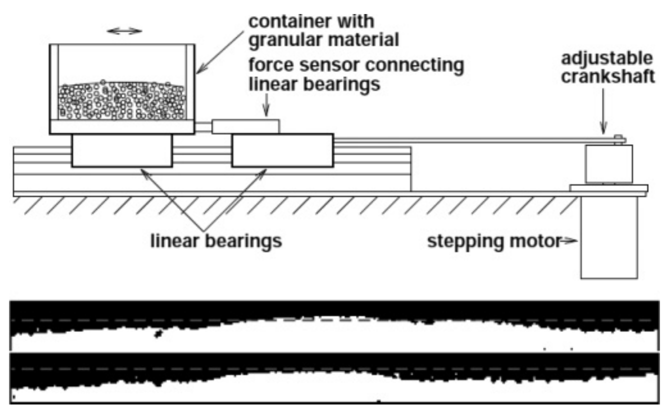

FIG. 1. Top: Experimental setup (see text for explanation). Bottom: Snapshots of the surface region of the material inside the container displaying about 1 cm $\times$ 6 cm (stretched photographically) of the topmost central part of the grains. The dashed reference line, located at a constant height above the bottom of the container, helps to visualize the extension of the inflation.

Figure 1 illustrates schematically our experimental setup. A rectangular container was mounted on a precisely balanced horizontal linear bearing (left in Fig. 1). A second bearing (middle) was driven by a stepping motor via a crankshaft with adjustable eccentricity. Both bearings were connected by a piezoelectric force sensor, which allows us to measure the driving force with 2 kHz time resolution. The motor was computer controlled so that, e.g., in precisely 2000 impulses its axis revolves once. This high angular-resolution provides quasisteady motion. The finite step size per computer signal does not influence the convective behavior. The entire mechanical device was fixed on an oscillation damping table. The container consists of transparent plastic material typically of size 6 cm $\times$ 10 cm, with the 10 cm oriented along the shaking direction [25]. It was illuminated from the top and monitored sideways by a videocamera perpendicular to the axis of oscillation. With this camera we monitored the dynamical processes in the container. To this end, the camera was leveled exactly at the vertical height of the granular material when at rest. At the bottom of Fig. 1 we illustrate two representative snapshots of 7 cm of the surface region.

Horizontally shaken containers display clear convective patterns [23]: Close to the walls the material flows downward, flowing upward in the central region. Flow patterns are considerably more complex under horizontal than under vertical shaking and, as mentioned, have not been completely understood yet [11–24]. By placing tracer particles of identical mechanical properties but different color on the upper surface and then shaking it was possible to observe a rather homogeneous spread of the tracers inside the material. This fact provides a clear indication that the entire granular material is involved in the convective motion underlying swelling.

III. RESULTS AND ANALYSIS

To characterize swelling quantitatively we recorded the temporal evolution of the granular height for selected sections of the material. The height evolution was recorded as a function of the forcing parameters, typically $A = 0.15$ cm and $f = 25\ \mathrm{s}^{-1}$. Figure 2(a) illustrates height variations measured for a section at the center of the container during 40s for different driving frequencies. We observe a small heap in the center whose height fluctuates from 2.3 mm (when collapsed) to 3.7 mm (when maximally swollen) with respect to the repose level. As discussed below, for the specified parameters the height fluctuates periodically with a period of about 2.2 s, which is 55 times the period of the driving oscillation. The height of the displayed regions is 0.4 cm each. At rest, the leveled material filling height was 2.5 cm. Figure 2(a) shows that the height varies with time for all frequencies. The height fluctuates irregularly with time when the driving frequency is either small ($f \lesssim 22\ \mathrm{s}^{-1}$) or large ($f \gtrsim 30\ \mathrm{s}^{-1}$). However, for $23\ \mathrm{s}^{-1} \lesssim f \lesssim 29\ \mathrm{s}^{-1}$ we observe a quite regular, almost periodic oscillation of the height at the center.

The periodic swelling behavior can be characterized by Fourier transforming the time-evolution of the height. Figure 2(b) shows such transform for series of 120 s, i.e., three times longer those shown in Fig. 2(a). For $f < 22\ \mathrm{s}^{-1}$ no characteristic frequency can be observed; i.e., the Fourier spectrum contains many frequencies. At $f = 22\ \mathrm{s}^{-1}$ there is a sharp transition into another regime, where we find a characteristic frequency of the height oscillation, as indicated by a peak in the Fourier spectrum. For high frequency, $f \gtrsim 29\ \mathrm{s}^{-1}$, the swelling amplitude becomes very small, and the effect vanishes. Hysteresis of the transition point has not been observed.

While the time scales of the dynamical phenomena observed under standard vertical shaking are always comparable to the period of the external forcing [26–29], for horizontal vibration we find a marked difference: Swelling displays much longer periodicities, which may be up to several hundred times larger than the period f^{-1} of the external drive. Thus, as far as we know, swelling is the first dynamical effect where the frequency of forcing decouples from the frequency of the induced phenomenon; i.e., horizontal shaking reveals an intrinsic time scale of the dynamics [30]. Due to the low frequencies involved, we do not observe any period-doubling scenario [28,29].

Viewing the container from top one observes that the oscillation of material height (Fig. 2) corresponds to a varying particle velocity at the upper surface. When the material is swelling there is an intensive flow, whereas the flow in the center of the surface comes almost to rest when the material is collapsed. A simple way to visualize this effect is to take a series of snapshots of the container viewing from the top using

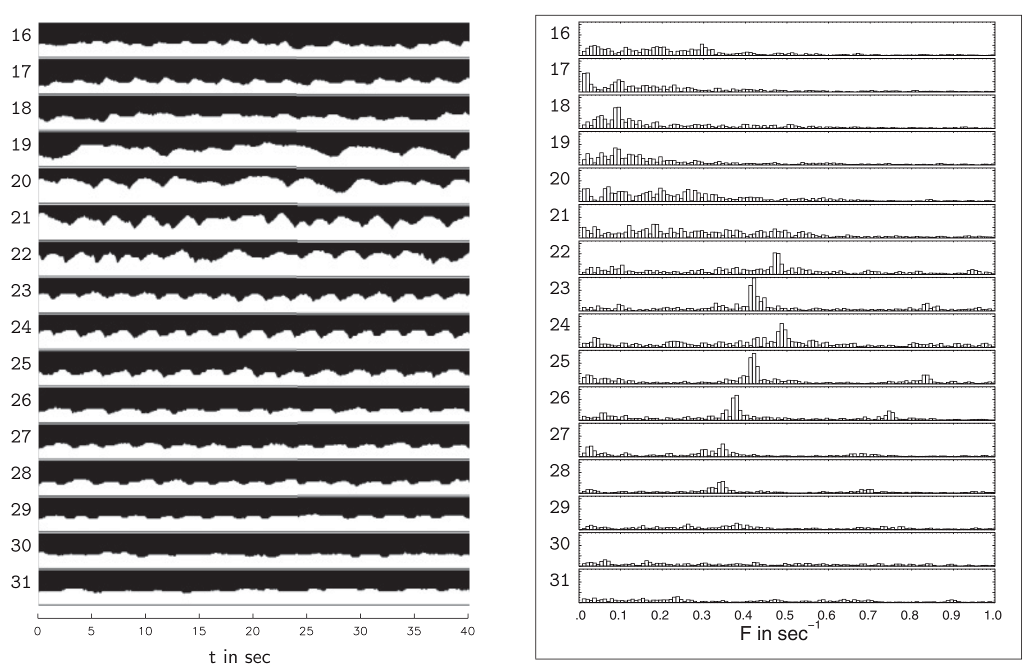

FIG. 2. Left: Height of the granular material in the middle of the container over time for driving frequencies f between 16 and 31 s^{-1}. The height of the granular material in each panel displays about 1 cm × 6 cm (stretched photographically) of the topmost central part of the grains. Right: Fourier transforms of the oscillation height seen on the left. Units of the vertical scales are arbitrary.

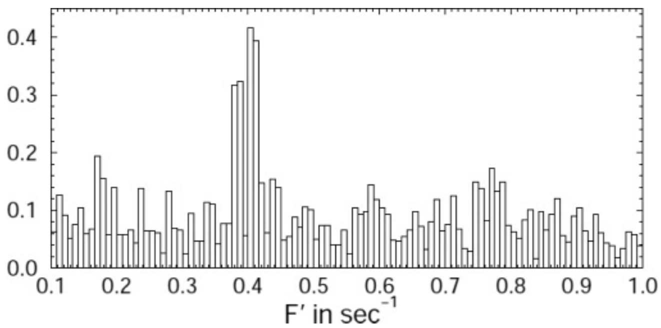

FIG. 3. Material flow at the surface (central area of size 6.5 cm × 2 cm) over time for $f = 25\ \mathrm{s}^{-1}$ (see text). The units of the vertical scale are arbitrary.

a digital camera. From these snapshots we derive "difference pictures" by subtracting the grayscale values of the pixels of two consecutive snapshots [31]. The average value ΔG of the pixel grayscale in such difference pictures provides a suitable measure of the flow at the surface. Figure 3 shows the Fourier transform of ΔG for a driving frequency of $f = 25\ \mathrm{s}^{-1}$. From this figure one sees that the variation of the surface flow has the same characteristic frequency as the height oscillations shown in Fig. 2. This fact indicates that swelling and convective motions in horizontally shaken containers are directly correlated effects.

From the force data $F(t)$ measured by the sensor we calculated the dissipated driving energy during the nth shaking period:

$$E_n = A \int_{n/f}^{(n+1)/f} F(t) \cos(2\pi f t)\, dt. \tag{1}$$

E_n is the energy dissipated during the nth period by the entire system, i.e., by the linear bearing and by the dissipative interaction of the grains in the container. The time series E_n can be analyzed by Fourier analysis similarly to the surface flow intensity and the height of the material. Figure 4 shows the Fourier transform of the dissipated energy for a range of driving frequencies f. It agrees very well with Fig. 2(b), indicating that the characteristic frequencies of height oscillation and of energy dissipation coincide. Low-frequency components exist in Figs. 2 and 4 but do not depend of the air pressure (see next paragraph). So far we have not been able to track down their origin. We will explore this issue elsewhere. In conjunction with the other experimental observations, this leads us to believe that the swelling effect is essentially connected to a time-dependent convection, namely, with a time-dependent material flow and energy dissipation inside the material.

Is air responsible for swelling? Air effects in granular materials are discussed in a classical work by Pak *et al.* [32]. One could imagine that the convective motion would "pump" air into the bulk of the material, leading to swelling. When the air content of the material reached a certain threshold an air bubble could escape, causing grain collapse. Such a mechanism could account for both swelling and the saw-tooth shape of the height over time curves (Fig. 2). However, reducing the pressure to $p = 50$ Pa, corresponding to a

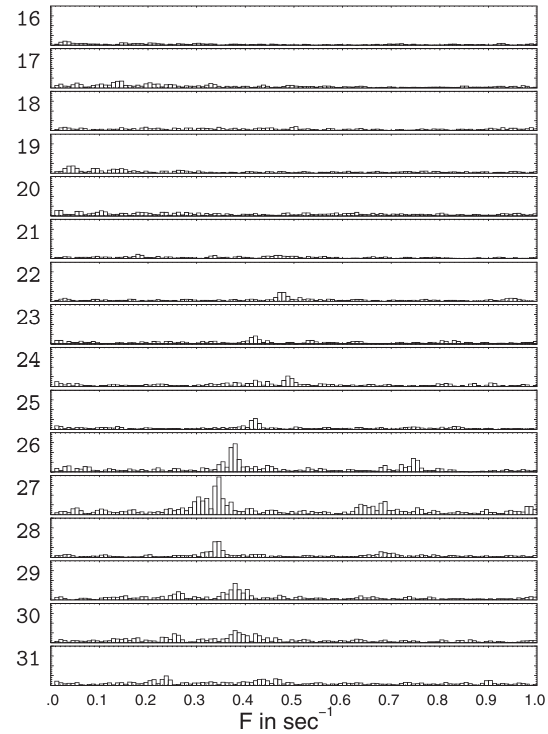

FIG. 4. Fourier spectra of the dissipated energy E for a range of frequencies, between 16 and 31 s^{-1}, and driving amplitude $A = 0.15$ cm. Vertical scales are in arbitrary units.

mean-free path of $\lambda_{\text{-}} \approx 130\ \mu$m, we observe swelling not to disappear. Further reduction to 10 Pa (mean-free path $\approx 650\ \mu$m) preserved swelling. Since λ_{air} is smaller than the typical free path of sand grains, we can therefore safely exclude air as the major motor of swelling.

Note that for lower pressures, as the mean-free path increases, it becomes much larger that the free path between grains, implying that air effects are small. An alternative dimensionless number to estimate this effect is to consider the ratio between the mass of air between grains and the mass of grains. The packing density of sand (random close packing) is about 0.64. Since in our case the material flows slowly, here the fraction of solid material is a bit below, e.g., 0.6. Thus, each volume element consists of 60% solid and 40% air. The material density of sand is about 1600 kg/m^3; therefore, the solid material density is 1600 kg/m^3/0.6 = 2666 kg/m^3. The density of air is $p/(R_s T)$, where the specific gas constant is $R_s = 287\,058$ J/(kg K) and the temperature is $\sim$293 K. Assuming a pressure of $p = 10$ Pa we obtain for the density 1.19×10^{-4} kg/m^3. Each volume element V contains a mass of solid material given by $m_{\mathrm{solid}} = V \times 0.6 \times 2666$ kg/m^3 and $m_{\mathrm{air}} = V \times 0.4 \times 1.19 \times 10^{-4}$ kg/m^3 of air. Thus, the ratio between the mass of air between grains and a grain is about $m_{\mathrm{air}}/m_{\mathrm{solid}} \simeq 3 \times 10^{-8}$, a rather small value, consistent with the conclusion of the previous paragraph.

As a function of the driving frequency f, the swelling frequency $F = F(f)$ can be reproduced experimentally within a few percent of accuracy for different filling heights. However, we were unable to correlate our experimental data using dimensionless numbers for many combinations of the standard parameters $A^\alpha \omega^\beta h^\gamma$, where h is the filling level and $\omega = 2\pi f$.

No correlation was found using the relative acceleration amplitude $A\omega^2/g$, or the amplitude of the velocity $A\omega$, so relevant in many other experiments [1–10]. Our observation that neither the acceleration amplitude nor the velocity amplitude is the only relevant dimensionless parameter for vibration-induced phenomena agrees with previous experiments (e.g., Refs. [28,29]) and simulations (e.g., Refs. [33,34]). For a one-dimensional system of viscoelastic particles it can be shown analytically that the dimensionless parameter is not even of the form $A^\alpha\omega^\beta$ but contains an additional time scale involving the size L of the system [see Eqs. (36) and (37) in Ref. [35]]. Thus, since even for a *one-dimensional* vibrated column the dimensionless parameter is a complicated combination of the shaking frequency, amplitude, and system size, the difficulty of uncovering relevant dimensionless parameters for our *three-dimensional* experiment should not be surprising.

The origin of recurrent swelling is admittedly not yet completely clear. Here we offer a tentative explanation based on the measurement of dissipated mechanical energy and surface flow: In horizontally shaken containers one observes convection [11–24], i.e., motion of the particles with respect to each other. To allow for macroscopic motion, the granular material has to be diluted below a certain density ρ_R below Reynolds dilatancy [36]. When the local density is below ρ_R the material can start to flow. The then starting convection in the container implies shear flow, which causes further dilution and fluidization (e.g., Ref. [37]). The increasing shear flow also implies an increasing dissipation of mechanical energy, which we observed in our measurements. When the shaken material swells according to Reynolds dilatancy, the material becomes diluted and, at the same time, less mechanically stable so that the top layers cannot be fully supported anymore by the diluted layers underneath. At a certain moment the material becomes diluted to an extent that it completely loses mechanical stability and collapses. The flow comes then almost to rest and the whole swelling process starts over again. Thus, we believe ourselves to be in the presence of two competing effects: First, shear tends to dilute the material, and, second, the material needs to remain mechanically stable. Their interplay can explain the observed volume oscillations.

Apart from the two different time scales discussed, due to the driving frequency and to the frequency of swelling, a third scale may be defined by the frequency of convection, i.e., by the typical time for a particle to execute one cycle of the convection. Using tracer particles we find this time to be on the order of minutes, meaning that the corresponding frequency also decouples from the frequency of swelling.

IV. CONCLUSIONS AND OUTLOOK

A number of interesting questions remain open, e.g., concerning the swelling dependence on container shape and size as well as on the details of grain properties and the amplitude of shaking. While specific characteristics of swelling depend on the details of the experiment, we observed swelling to be a rather robust effect over wide ranges of material and forcing parameters. An additional challenge is to perform a simulation, taking into account the specific nature of the lateral forces acting on the grains, and capable of reproducing the recurrent swelling. Since swelling depends on the specific three-dimensional character of the force field, we do not expect two-dimensional simulations to be able to reproduce it.

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