Comment on "Rapidly Converging Bounds for the Ground-State Energy of Hydrogenic Atoms in Superstrong Magnetic Fields"

In a recent Letter Handy *et al.*[1] applied the *moment method* to the problem of a hydrogen atom in superstrong magnetic fields, generating lower and upper bounds to the eigenenergy of the ground state. Their paper contains critical remarks regarding calculations by previous authors. Some assertions in their paper seem to imply that the moment method has once and forever solved the problem that is frequently quoted in literature as the "last" unsolved problem in atomic physics.

The purpose of this Comment is to point out that although lower and upper energy bounds are interesting pieces of information, accurate results on the quadratic Zeeman eigenspectra have been obtained in recent years by several groups interested in the investigation of *quantum chaos* in a realistic physical system (see Refs. 2–4, and several references therein). Accurate eigenvalues are known not only for the ground state but also for hundreds of excited states.[2–4] Handy *et al.* compare their results with those of LeGuillou and Zinn-Justin[5] while [for reasons clearly recognizable from Table 2 of Ref. 6(b) below] a more realistic comparison can be made by considering the eigenvalues reported by Wunner and Ruder.[7] The *high-field* limit is also very well represented in the calculations of Baye and Vincke.[8] Lower and upper bounds on an extended but finite field interval have been recently provided by Liu and Starace.[9] A consistent representation of the whole field interval is provided by the simple basis introduced by one of us.[6]

TABLE I. Binding energies $E_B = \frac{1}{2}\gamma - E$ for a magnetized hydrogen atom. The energies E taken from variational calculations described in Refs. 10 and 11 are accurate upper bounds. The best upper bound provides the highest binding energy. γ is the field strength in units of 2.35×10^5 T.

γ	Lower bound[a]	Upper bound[a]	RWHR[b]	RGG[c]
2	1.0222138	1.022 2142	1.022214	1.022214
20	2.215325	2.215450	2.215398	2.215398
200	4.710	4.740	4.72655	4.727134
300	5.34	5.39	5.36030	5.360799
1000	7.55	7.85	7.66205	7.662388
10000				14.140629

[a] Reference 1.

[b] Reference 10.

[c] Reference 11.

Table I presents a comparison of upper and lower bounds obtained by Handy *et al.* with two of the best results from the literature obtained from variational calculations. From this table we conclude that although the results of Handy *et al.* are of methodological value, a final solution to the "last" problem is still missing: The moment method provides energy bounds with one or two significant figures over an extended but finite interval whereas two elaborate variational calculations (using flexible basis and providing accurate upper bounds) already agree to four significant figures.

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