

Nonuniformities in the Angle of Repose and Packing Fraction of Large Heaps of Granular Particles: Supplementary Material

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THE RATE OF GROWTH OF THE HEAP

Let the quantities r, S, h, δ define the geometry of the heap as illustrated in Fig. 1. We wish to find a relation between the angle of repose δ and the change in the area of the flat surface dr/dh of the heap under the simplifying assumptions that the heap has constant density, a single angle of repose, grows symmetrically, and that the material added to lateral surface has constant thickness.

From Fig. 1 it follows that $s = x \sin(\gamma - \delta)$ and $\Delta h = x \sin \gamma$ so that, eliminating x , we get:

$$\frac{s}{\Delta h} = \frac{\sin(\gamma - \delta)}{\sin \gamma}. \quad (1)$$

This relation may be also obtained as follows. The constant density assumption implies that $V = V_t + V_f$, where V denotes the total volume of deposited material at time Δt , V_t the volume deposited on the *tilted* surface, and V_f is the volume on the *flat* surface. When Δt is small, the volume contained between flat and tilted surface becomes negligible and the particles being deposited end up located either on the flat surface of radius $r < S$ or inside the lateral tilted surface. Since particles are dropped as a homogeneous rain through the area source, the number of particles that fall onto the flat surface of the heap in time Δt is $N_f = \pi r^2 F \Delta t$. Here F is the particle flux through the source, i.e. the number of particles that pass through the unit area of the source in unit time. Since the total number of particles passing through the whole source in an interval Δt is $N = \pi S^2 F \Delta t$, we find that the number of particles that end up on the tilted surface is $N_t = N - N_f = \pi(S^2 - r^2)F \Delta t$.

For constant packing fraction n , N_f contributes to the change of Δh ,

$$\pi r^2 F \Delta t = n \pi r^2 \Delta h \quad (2)$$

and N_t contributes to s ,

$$\pi(S^2 - r^2)F \Delta t = n s \pi(L^2 - r^2)/\cos \delta, \quad (3)$$

where $\pi(L^2 - r^2)/\cos \delta$ is the surface of the frustum. From Eqs. (2) and (3) we obtain

$$\frac{s}{\Delta h} = \frac{S^2 - r^2}{L^2 - r^2} \cos \delta. \quad (4)$$

Comparing Eqs. (1) and (4) one readily sees that

$$\frac{\sin(\gamma - \delta)}{\sin \gamma} = \frac{S^2 - r^2}{L^2 - r^2} \cos \delta \quad (5)$$

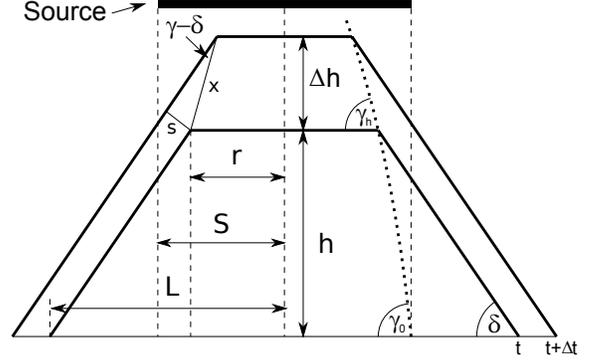


FIG. 1. Schematic view of a central section of the symmetrically growing heap, characterized by the h -dependent angle $\gamma \equiv \gamma_h$, drawn for times t and $t + \Delta t$ (ticker lines). The random “rain” of particles originates from a circular source with radius S . For clarity, the distance between the source and the heap was greatly reduced here.

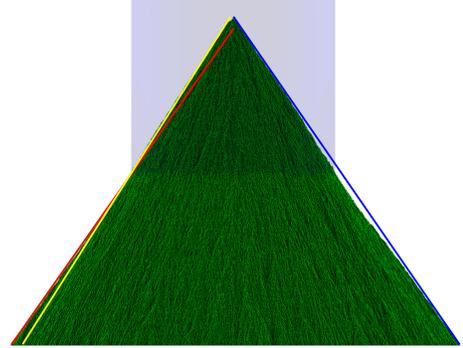


FIG. 2. (Color online) The two angles of repose, made visible by three distinct tangent lines to the heap containing 10×10^6 particles. The rectangular box on the top indicates the region where particles are dropped from the top. Outside the dropping zone the straight line is tangent to the surface while inside the dropping zone it lies below the surface.

Now, using the addition formula for the sin function and the relation $L = r + h \cot \delta$ extracted from the figure, we obtain:

$$\tan \gamma = \frac{(r + h \cot \delta)^2 - r^2}{(r + h \cot \delta)^2 - S^2} \tan \delta. \quad (6)$$

Thus, since $dr/dh = -\cot \gamma$, we obtain a differential

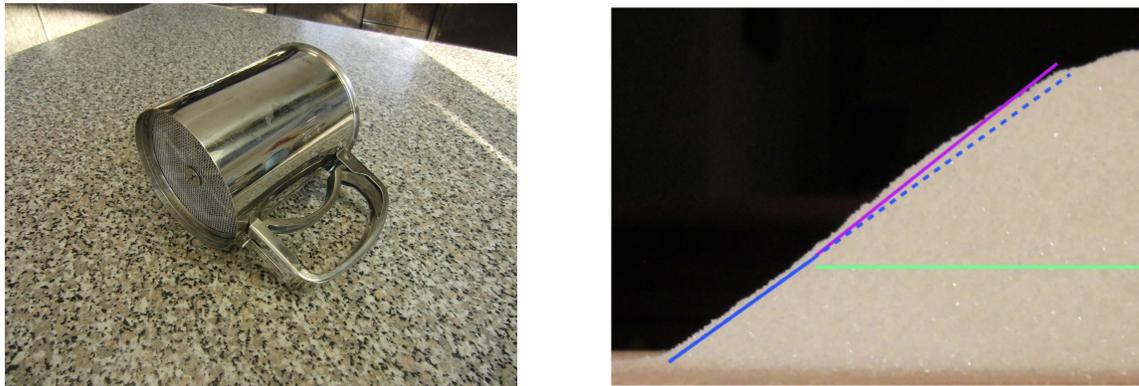


FIG. 3. (Color online) **Left**: Kitchen sieve used to produce a circular rain of sugar grains. **Right**: Pile of sugar grains displaying two angles of repose. The horizontal green line marks the extension of the rain (dropping zone). For clarity, we show just one side of the symmetric heap obtained.

equation for $r(h)$, the function describing how the flat surface shrinks in time:

$$\frac{dr}{dh} = - \frac{(r + h \cot \delta)^2 - S^2}{(r + h \cot \delta)^2 - r^2} \cot \delta. \quad (7)$$

The proper initial condition is $r(h = 0) = S$ and the negative sign indicates that the area shrinks from its initial value. Of course, the solution of Eq. (7) is only physically meaningful as long as $r(h) \geq 0$.

Figure 2 illustrates the two angles of repose for a pile obtained by numerically depositing 10×10^6 particles.

Lastly, Fig. 3 shows a simple way to demonstrate experimentally the distinct angles of repose. The figure illustrates the pair of angles for a pile obtained from a simple “kitchen table” experiment in which we “rained” about 0.7 kg of ordinary fine grained sugar through an ordinary kitchen sieve used to spread sugar on cakes, with 8 cm of diameter (Fig. 3 left). The result is shown in the rightmost panel of Fig. 3, where one may distinguish both angles with the help of the auxiliary lines intended to guide the eye.